ELEC 102 Electronic Circuits Hour Exam II Fall 2000

November 14, 2000 Time: 6:30:- 7:45 pm

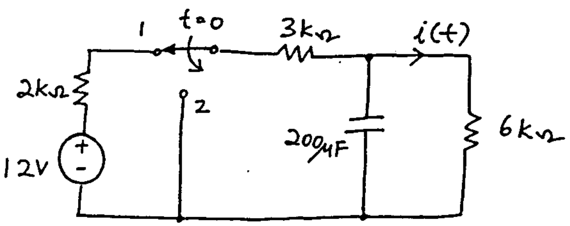
Name: Solution Department: Seat number:

Student ID: Sample Email address:

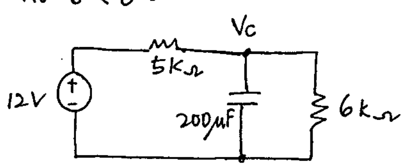
Note: Show all your steps, method chosen to solve the problem, and formulas used as clearly as possible to maximize your credits. You may use the back side of the exam papers for your rough work. Allocate time to each question proportional to the points assigned to each problem. Make sure you answer all the questions asked in each problem.

Questions	Q1	Q2	Q3	Q4	Q5	Q6	Q7
Points							

1) In the circuit shown below, the switch is moved to position 2 at t = 0 after being in position 1 for a long time. Find i(t) for t > 0.(15 points)

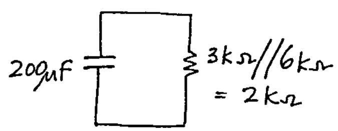


At $t<0$:



$$V_c(0) = 12\left(\frac{6k}{5k+6k}\right) = \frac{72}{11}\ V$$

As $t \to \infty$: $V_c(\infty) = 0V$

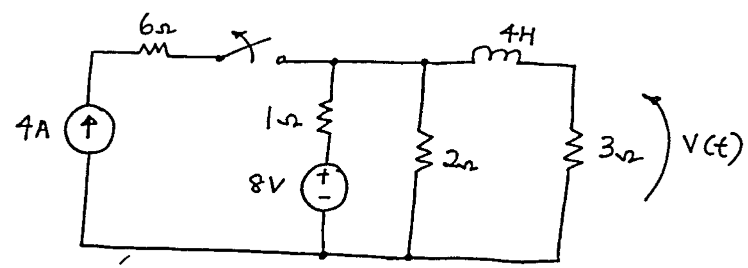


$$\therefore V_c(t) = V_c(0)e^{-t/\tau} = \frac{72}{11}e^{-t/0.4}\ V$$

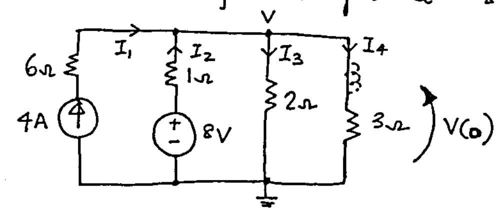
$$\tau = RC = 2K(200\mu) = 0.4s$$

$$\therefore i(t) = \frac{V_c(t)}{6K} = \frac{12}{11}e^{-t/0.4}\ mA = 1.09\,e^{-2.5t}\ mA$$

2) The switch in the following opens at t =0, find v(t) for t > 0.(20 points)



At $t<0$: (for a long time — steady state)



$$I_1 + I_2 = I_3 + I_4$$

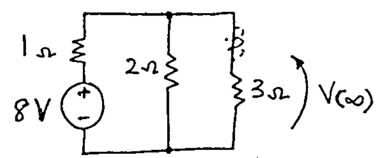
$$4 + \frac{8-V}{1} = \frac{V}{2} + \frac{V}{3}$$

$$24 + 48 - 6V = 3V + 2V$$

$$V = \frac{72}{11}V$$

$$\therefore V(0) = \frac{72}{11}V = 6.545\ V$$

As $t \to \infty$,

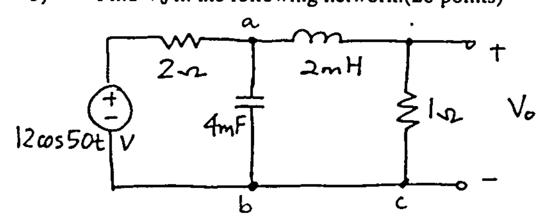


$$\tau = \frac{L}{R} = \frac{4}{(1//2)+3} = \frac{12}{11}s = 1.09s.$$

$$V(\infty) = \frac{2//3}{1+2//3}\times 8 = \frac{6}{5}\times\frac{5}{11}\times 8 = \frac{48}{11}V = 4.36\ V$$

$$V(t) = V(\infty) + [V(0)-V(\infty)]e^{-t/1.09} = \frac{48}{11} + \frac{24}{11}e^{-t/1.09}\ V$$

3) Find V_0 in the following network.(20 points)



$$12\cos 50t \to 12\angle 0^\circ\ V$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j50\times 4m} = -j5\ \Omega$$

$$Z_L = j\omega L = j50\times 2m = j0.1\ \Omega.$$

$$V_0 = \frac{1}{1+j0.1}V_{ac}$$

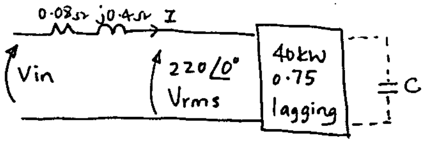
$$V_{ac} = V_{ab} = \frac{-j5//(1+j0.1)}{2+(1+j0.1)//-j5}\times 12\angle 0^\circ$$

$$V_0 = \frac{1}{\cancel{(1+j0.1)}}\times\frac{(-j5)\cancel{(1+j0.1)}}{(1-j4.9)\left[2+\frac{(-j5)(1+j0.1)}{(1-j4.9)}\right]}\times 12$$

$$= \frac{-j60}{2-j9.8+0.5-j5} = \frac{60\angle -90^\circ}{2.5-j14.8} = \frac{60\angle -90^\circ}{15\angle -80.4^\circ} = 4\angle -9.6^\circ\ V$$

$$V_0(t) = 4\cos(50t - 9.6^\circ)\ V$$

4) A lagging load with a power factor of 0.75 consumes 40kW of power. The load voltage is 220 $\angle 0°$ V_{rms} at 50 Hz. The impedance of the line is 0.08 + j0.4 Ω. Find the voltage and power factor at the input of the line. Determine the value of the component needed to improve the PF to 1. (30 points)



$P_{load} = 40\,kW = V I \cos\theta_Z$

$I = \frac{40K}{220 \times 0.75} \angle -\cos^{-1} 0.75$

$= 242.4 \angle -41.4°$ A

$V_{in} = 220 + I(0.08 + j0.4)$

$= 220 + 242.4 \angle -41.4° \times 0.408 \angle 78.7°$

$= 220 + 98.9 \angle 37.3°$

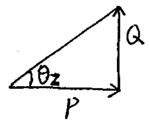
$= 220 + 78.7 + j59.9$

$= 298.7 + j59.9 = \underline{304.65 \angle 11.3°\ V_{rms}}$

Power factor at line input

$\cos(\theta_v - \theta_i) = \cos(11.3 - (-41.4))$

$= \cos 52.7° = \underline{0.61}$



$\theta_Z = 41.4°$

$f = 50$ Hz

$Q_{load} = 40K \tan(41.4°)$

$= 35.26$ KVAR

To correct the PF to 1, We need to connect a capacitor across the load.

$Q_{new} = 0$ VAR $= Q_{load} + Q_C$

$Q_C = -j35.26$ KVAR

$Q_C = -jV^2 \omega C$

$\therefore C = \frac{35.26K}{220^2 (2\pi 50)}$

$= 2.32$ mF

5) A series LRC circuit has a resonant frequency of 2000 rad/s and a bandwidth of 100 rad/s. Determine the values of L and C if R is 5 Ω. (15 points)

$\omega_0 = 2000$ rad/s $\quad BW = 100$ rad/s

$BW = \frac{\omega_0}{Q}$

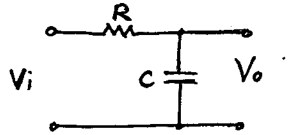
$Q = \frac{\omega_0}{BW} = \frac{2000}{100} = 20$

By definition: $Q = \frac{\text{Reactive Power}}{\text{Resistive Power}} = \frac{\omega L}{R} \Rightarrow L = \frac{20 \times 5}{2000} = 0.05$ H

$= \frac{\frac{1}{\omega C}}{R} \Rightarrow C = \frac{1}{2000 \times 20 \times 5} = 5\,\mu F$

6) Given a 5k and a 10kΩ resistors, some capacitors and inductors, design a low pass filter with a cutoff frequency of 10 rad/s and a high pass filter with a cutoff frequency of 100rad/s by specifying the values of the C and/or L. (15 points)

Low pass filter:



$\frac{V_o}{V_i} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega CR}$

At cutoff frequency,

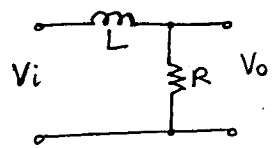
$\left|\frac{V_o}{V_i}\right| = \frac{1}{\sqrt{2}} \Rightarrow \omega CR = 1$

$\omega = \frac{1}{CR} = 10$ rad/s

$\Rightarrow C = \frac{1}{10 \times 10K} = 10\,\mu F$

If $R = 10K\Omega$, $C = 10\,\mu F$

$R = 5k\Omega$, $C = 20\,\mu F$

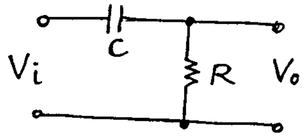


$\frac{V_o}{V_i} = \frac{R}{R + j\omega L} = \frac{1}{1 + j\omega\frac{L}{R}}$

At cut-off frequency, $\omega = \frac{R}{L} \Rightarrow$ If $R = 10K\Omega$, $L = 1KH$.

If $R = 5k\Omega$, $L = 500H$

High pass filter:



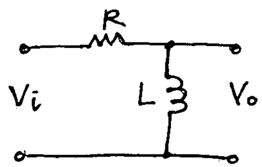
$\frac{V_o}{V_i} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{1}{1 - \frac{j}{\omega CR}}$

At corner frequency, $\left|\frac{V_o}{V_i}\right| = \frac{1}{\sqrt{2}}$

$\Rightarrow \omega = \frac{1}{CR} = 100$ rad/s

If $R = 10k\Omega$, $C = 1\,\mu F$

$R = 5k\Omega$, $C = 2\,\mu F$



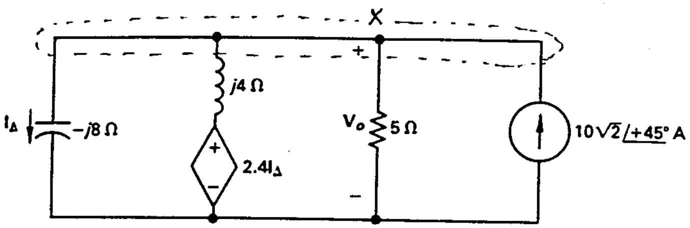
$\frac{V_o}{V_i} = \frac{j\omega L}{R + j\omega L} = \frac{1}{1 + j\frac{R}{\omega L}}$

$\omega = \frac{R}{L}$ at 3dB cut-off.

If $R = 10k\Omega$, $L = 100H$

$R = 5k\Omega$, $L = 50H$

7) Use the node-voltage method to find the phasor voltage V0 in the circuit below. Express the voltage in both polar and rectangular form. (20 points)



Applying KCL at node X;

$$\frac{\mathbf{V}_o}{-j8}+\frac{\mathbf{V}_o-2.4\mathbf{I}_\Delta}{j4}+\frac{\mathbf{V}_o}{5}=10\sqrt{2}\underline{/+45^\circ}$$

$$j5\mathbf{V}_o-j10(\mathbf{V}_o-2.4\mathbf{I}_\Delta)+8\mathbf{V}_o=400\sqrt{2}\underline{/+45^\circ}$$

$$\mathbf{V}_o[j5-j10+8]+j24\frac{\mathbf{V}_o}{-j8}=400\sqrt{2}\underline{/+45^\circ}$$

$$\mathbf{V}_o(8-j5-3)=400\sqrt{2}\underline{/+45^\circ}$$

$$\mathbf{V}_o(5-j5)=400\sqrt{2}\underline{/+45^\circ}$$

$$\mathbf{V}_o=\frac{400\sqrt{2}\underline{/+45^\circ}}{5\sqrt{2}\underline{/-45^\circ}}=80\underline{/90^\circ}\,\mathrm{V}$$

$$\mathbf{V}_o=0+j80\,\mathrm{V}$$