Math132 L1 & L2 Discrete Structure: Final Exam
Dept of Math, HKUST, Fall 1998

Name:__________________ Class Section:__________________

ID No.__________________ Tutorial Section:__________________

Problem	1 (27)	2 (15)	3 (10)	4 (10)	5 (10)	6 (10)	7 (10)	8 (8)	Total (100)
Score									

1. (27, each 3 pts) Fill in the blank space for each of the following problems.

   (a) The number of arrangements of r elements from a source of n elements is _____.

   (b) The number of subsets of r elements from a set of n elements is _____.

   (c) The number of ways of selecting r elements from a source of n elements with repetition is _____.

   (d) Every tree with more than one vertex has at least _____ leaves.

   (e) The number of labeled subgraphs with n vertices of the complete graph K_n is _____.

   (f) The number cycles of length 4 in the complete bipartite graph $K_{m,n}$ is _______.

   (g) The six letters A, A, B, B, B, and C can be arranged in _______ ways.

   (h) The height of a binary tree with n vertices is bounded by _______.

   (i) For set A and B, if $A| = m$ and $B| = n$, then the number of injective functions from A to B is _______.

2. (15, each 3 pts) Please circle one and only one which is correct.

   (a) i. The implication $\rightarrow$ can not be represented by $\neg$, $\vee$, and $\wedge$.
      ii. The negation $\neg$ can not be represented by $\vee$, $\wedge$, and $\rightarrow$.
      iii. The disjunction $\vee$ can not be represented by $\neg$, $\wedge$, and $\rightarrow$.
      iv. The conjunction $\wedge$ can not be represented by $\neg$, $\vee$, and $\rightarrow$.
      v. None of above.

   (b) i. The universal quantifier $\forall$ can not be represented by $\exists$ and $\neg$.
      ii. The existential quantifier $\exists$ can not be represented by $\forall$ and $\neg$.
      iii. For the sentences $p(k)$ with $k \geq 1$, $p(1) \wedge (\forall k \geq 1, p(k) \rightarrow p(k+1)) \rightarrow (\forall k \geq 1, p(k))$ is a tautology.
      iv. None of above.

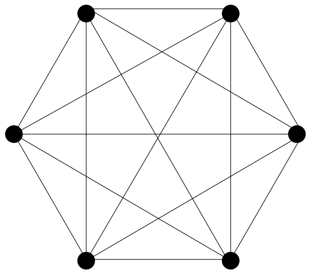
   (c) Let A and B be finite sets and let $f : A \longrightarrow B$ be a function.
      i. If f is injective, then f may not be surjective.
      ii. If f is is surjective, then f may not be injective.
      iii. If f is injective or surjective, then f is a one-to-one correspondence.
      iv. None of above.

3. (10=5+5 pts)

(a) Find the solution for the difference equation $u_n + 6u_{n-1} + 9u_{n-2} = 0$ with $u_0 = u_1 = 2$.

(b) Find the explicit expression for the sequence $f(n) = 4f(\frac{n}{4}) + 8$ with $f(1) = 8$, where $n = 4^k$, $k = 1, 2, \cdots$.

4. (10 pts) Use Euler's formula for planar graphs to show that the following graph is not planar.



5. (10=3+3+4 pts) Let $K_{m,n}$ denote the complete bipartite graph $K_{m,n}$, i.e., $V(K_{m,n})$ can be divided into two disjoint union of two sets V_1, V_2 with $|V_1| = m$, $V_2 = n$, and every vertex of V_1 is adjacent with every vertex of V_2.

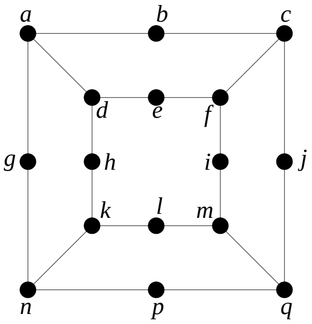
(a) For what values of m and n that $K_{m,n}$ has an Euler tour?

(b) For what values of m and n that $K_{m,n}$ has a Hamilton cycle?

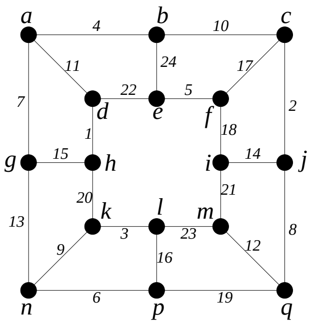
(c) For what values of m and n that $K_{m,n}$ is planar?

6. (10=6+4 pts)

(a) (6 pts) Use Depth-First Search and Breadth-First Search methods to find a spanning rooted tree with root d for the following graph. The order of the vertices is the alphabetical order.



(b) (4 pts) Find a minimal spanning tree for the following weighted graph.



7. (10=4+3+3 pts) Let A^∞ denote the set of infinite sequences of numbers 0 and 1. Let R be a relation on A^∞, defined by aRb if and only if the two infinite sequences a and b are different at only finite number of positions.

   (a) Show that R is an equivalence relation.

   (b) For each equivalence class of R, is it a finite set, infinite countable set, or uncountable set? Please circle one (no need reason): FINITE, INFINITE COUNTABLE, UNCOUNTABLE.

   (c) Is the set of equivalence classes countable? Please circle one (no need reason): YES or NO.

8. (8 pts) Find the language $L(M)$ of the following deterministic finite automaton M, where the starting state is at s_0 and the final states are s_3 and s_4.