Math132 L1 Discrete Structure: Final Exam

Dept of Math, HKUST, Fall 1999; Dec. 14, 8:30AM-11:30AM

Name:____________________ Class Section:____________________

ID No.____________________ Tutorial Section:____________________

Problem	1 (16)	2 (16)	3 (10)	4 (10)	5 (15)	6 (14)	7 (10)	8 (9)	Total (100)
Score									

1. For the following multiple choice questions, please choose one and only one answer. (4 pts each)

   (a) Which of the following is an infinite countable set?

   i. The set of real numbers.
   ii. The set of complex numbers.
   iii. The set of irrational numbers.
   iv. The set of prime numbers.
   v. The set of all numbers appeared somewhere in a book in our world at the moment.

   (b) A subset S of the set $\{\neg, \wedge, \vee, \rightarrow, \leftrightarrow\}$ is said to be *adequate* if every symbolic logic sentence without quantifiers can be expressed by the symbols of S. Which of the following sets is NOT adequate?

   i. $\{\wedge, \vee, \rightarrow, \leftrightarrow\}$
   ii. $\{\neg, \vee, \rightarrow, \leftrightarrow\}$
   iii. $\{\neg, \wedge, \rightarrow, \leftrightarrow\}$
   iv. $\{\neg, \wedge, \vee, \leftrightarrow\}$
   v. $\{\neg, \wedge, \vee, \rightarrow\}$

   (c) Let $G = (V, E)$ be a simple graph, i.e., G has no loops and multiple edges. Which of the following is correct?

   i. If G has no cycles, then G is a tree.
   ii. If $|E| = |V| - 1$, then G is a tree.
   iii. If there exists a unique path between every pair of distinct vertices, then G is a tree.
   iv. G has at least two leaves.
   v. None of the above.

   (d) Which one of the following is the best lower bound for the height of an m-ary ($m \geq 2$) tree with n leaves?

   i. $\log_n m$
   ii. $\log_m n$
   iii. $n \log_m n$
   iv. n
   v. None of the above.

2. (4 pts each; no need reason)

   (a) What is the chromatic polynomial of a tree with n (≥ 2) vertices?
   Answer:

   (b) How many ways are there to put 10 identical balls to 3 distinct boxes so that each box contains at least one ball?
   Answer:

   (c) What is the probability of selecting 5 cards from a poker of 52 cards so that there are exactly 3 red cards and 2 black cards?
   Answer:

   (d) Given 3 national flags, 3 HKSAR flags, and 2 UST flags. In how many ways can the 5 flags be put on a single pole vertically?
   Answer:

3. (a) Find the solution of the difference equation $x_{n+2} = 10x_{n+1} - 25x_n$, where $n \geq 0$ and $x_0 = 1$ and $x_1 = 2$. (5 pts)

   (b) Let $f(n) = 3f(n/2) + 3$ and $f(1) = 3$, where $n = 2^k$, $k = 1, 2, \cdots$. Find $f(n)$. (5 pts)

4. Let $f, g : \mathbf{R} - \{0, -1\} \longrightarrow \mathbf{R} - \{0, -1\}$ be functions defined by $f(x) = \frac{1}{x}$ and $g(x) = -(x+1)$ respectively.

   (a) Choose one and only one of the following: (3 pts)

      i. Only f is a one-to-one correspondence;

      ii. Only g is a one-to-one correspondence;

      iii. Both f and g are one-to-one correspondences;

      iv. None of them is a one-to-one correspondence.

   (b) Is the set $F = \{h_1 \circ h_2 \circ \cdots \circ h_k : h_1, h_2, \cdots, h_k$ are either f or $g, k \geq 1\}$ a finite set? If so, list all members of F. (7 pts)

5. Let $V = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Let R be a binary relation on V defined by aRb if and only if $1 \leq |a-b| \leq 2$. (3 pts each)

   (a) Is R a symmetric relation on V? If so, draw a (undirected) graph G for the relation R.

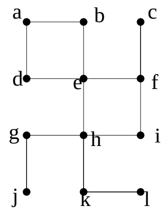
   (b) Does G have an Euler circuit or Euler path? If so, find an Euler circuit or Euler path.

   (c) Does G have an Hamilton cycle? If so, find a Hamilton cycle.

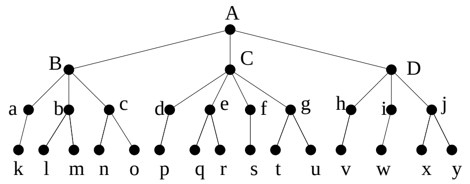
   (d) Draw the (undirected) graph for the relation $R \circ R$.

   (e) Let $\sim$ be a binary relation on V defined by $a \sim b$ if and only if $a - b$ is a multiple of 4. Is $\sim$ an equivalence relation on V? If so, find the set $V/\sim$ of equivalence classes and draw the quotient graph $G/\sim$.

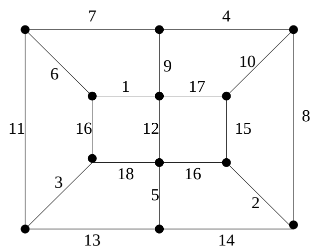
6. (a) Use both Depth-First Search and Breadth-First Search methods to find a spanning rooted tree with root a for the following graph. (6 pts)



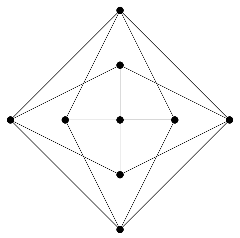
(b) Apply Preorder and Postorder methods to visit the vertices of the rooted tree. (4 pts)



(c) Find a minimal spanning tree of the weighted graph. (4 pts)



7. Apply Euler's formula for planar graphs to show that the following graph is not planar. (10 pts)



8. Design a Deterministic Finite Automaton M on alphabet set $\Sigma = \{a, b\}$ that accepts all strings which begin with aa and end with bb. (9 pts)