**Math132 L1 Discrete Structure: Midterm Exam**
Dept of Math, HKUST, Fall 1998

Name:____________________ Class Section:____________________

ID No.____________________ Tutorial Section:____________________

| Problem | No.1(10) | No.2(10) | No.3(15) | No.4(10) | No.5(10) | No.6(15) | No.7(10) | Total |
|---|---|---|---|---|---|---|---|---|
| Score | | | | | | | | |

1. Let $A$ and $B$ be sets. Please circle "Yes" or "No" for the following statements: (10 pts, each 2 pts)

   (a) $\overline{A \cup B} = \overline{A} \cup \overline{B}$ Yes or No

   (b) $A \cup \overline{B} = \overline{A} \cap B$ Yes or No

   (c) The set of rational numbers is uncountable. Yes or No

   (d) The set of real numbers between 0 and 0.000000000001 is countable. Yes or No

   (e) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ Yes or No

2. Let $p$, $q$ and $r$ be statements. Please circle "Yes" or "No" for the following statements: (10 pts, each 2 pts)

   (a) $(p \to q) \leftrightarrow (\neg p \vee q)$ is a tautology. Yes or No

   (b) $p \wedge (p \to q) \to q$ is a tautology. Yes or No

   (c) $(\forall x\ P(x)) \leftrightarrow \neg(\exists x\ \neg P(x))$ is a tautology. Yes or No

   (d) $(\forall k\ P(k) \to P(k+1)) \leftrightarrow (\forall k\ P(k))$ is a tautology. Yes or No

   (e) $[((\exists k\ p(k)) \to (\forall k\ q(k))] \leftrightarrow [\forall k\ p(k) \to q(k)]$. Yes or No

3. Find all integer solutions for the following equation

$$98x + 668y = 58$$

(15 pts)

4. Little John got 20 HK dollar coins, all of them are the same $1 coin. He has eight different pockets. In how many ways John can put his 20 coins into his eight pockets? (10 pts)

5. Find the probability of selecting eight cards from a poker of 52 cards so that there are exact 3 black cards and exact 5 red cards? (10 pts)

6. How many arrangements are there for the letters $\{A, B, C, D, E\}$ so that A is not in the 1st position, B not in the 2nd position, C not in the 3rd position, D not in the 4th position, and E not in the 5th position? (15 pts)

7. Show that among the integer numbers from 250 to 325, there are at least 9 numbers to which the difference of any two is a multiple of 8. (10 pts)