The Hong Kong University of Science & Technology

Department of Computer Science

COMP 271, Section 2: Design and Analysis of Algorithms

final examination

Date: 12:30 - 3:30 p.m., 12 December, 1998.

Venue: LG1 Sports Hall Lobby

Student name: ______________________________

Student ID: ______________________________

e_mail: ______________________________

Question	*Grade*
1	/24 pts
2	/12 pts
3	/8 pts
4	/12 pts
5	/8 pts
Total	/64 pts

<u>Problem 1</u>: Answer the following short questions.

(a) Mr. Careless claims that the running time of depth-first search on a graph G can always be done in $O(|V| + |E|)$ time, which is independent to the input graph representation. Discuss his claim.

(b) What is topological sort? Will the topological ordering of a directed acyclic graph be unique? Give an example to illustrate your answer.

(c) During the operation of Dijkstra's algorithm, we will perform relaxation process. What is the meaning of relaxation?

(d) Discuss how some real-life problem that can be modeled as minimum spanning tree problem.

(e) Discuss the difference between dynamic programming and divide and conquer algorithms.

(f) Does $P \subseteq NP$? Argue your answer briefly.

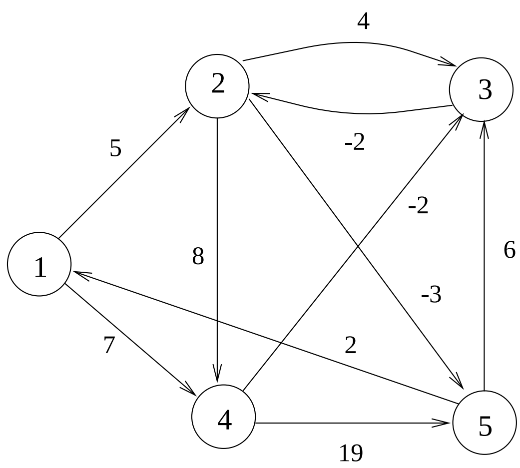
(g) What is Cook's theorem? What is the importance of Cook's theorem?

(h) Give the definitions of intractable problems and NP-Complete problems, compare the difference between them.

Problem 2:

(a) What is the difference between single-source shortest-path problem and all-pairs shortest-path problem?

(b) Given the following graph $G = (V, E)$, start with source 1, trace the actions of Dijkstra's algorithm. For each iteration, write down the d value of each vertex.



(c) For the same graph, trace the actions of Floyd-Warshall algorithm, show the matrix $D^{(k)}$ during each iteration.

(d) Compare your answers in part (b) and part (c), does the Dijkstra's algorithm correctly find the shortest paths? Why?

Problem 3:

(a) What is the meaning of prefix-free code? Why is it so important to ensure the property of the prefix-free codes?

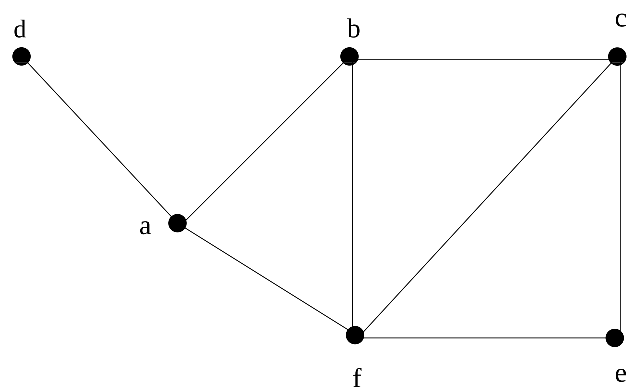
(b) Huffman invented a greedy algorithm to construct an optimal prefix-free code called Huffman code. What is being optimized in Huffman coding procedure? Argue briefly why Huffman code is prefix-free.

(c) Build an optimal Huffman tree for the following seven characters with the frequencies specified in parentheses: A(440), E(400), O(300), U(250), F(150), R(100), T(50).

Problem 4:

An independent set of a graph $G = (V, E)$ is a subset $V' \subseteq V$ of vertices such that each edge in E is incident in at most one vertex in V'. The optimization version of the independent set problem is to find a maximum-size independent set of G.

(a) Find the maximum-size independent set of the following graph.



(b) What is the related decision version for the independent set problem?

(c) Prove that the independent set problem is in NP.

(d) Prove that the independent set problem is NP-complete. You need not to argue the correctness of the reduction.

(*Hint* : Reduce from the clique problem.)

Problem 5:

Give an $O(n^2)$ dynamic programming algorithm which finds the longest monotonically increasing subsequence of a sequence of n numbers. Your algorithm should be self-contained, i.e., it should not call any algorithm as a subroutine. Analyze the running time of your algorithm.

In other words, the algorithm should find the largest subset of the sequence such that each successive number is greater than its predecessor. For the input sequence $< 1, 5, 2, 7, 3, 6 >$, the algorithm should produce $< 1, 2, 3, 6 >$.

(You need not to argue the correctness of your algorithm.)