The Hong Kong University of Science & Technology
Department of Computer Science

COMP271: Design & Analysis of Algorithms
Final Examination
Fall 1999

Problem 1 (10 pts)

Solve the following recurrence equation:

$$T(n) = \begin{cases} 1, & if \ \ n = 1 \\ 4T(n/2) + n, & if \ \ n > 1 \end{cases}$$

You can use any method to solve it.

Answer: Very easy $O(n^2)$.

Problem 2 (13 pts)

(a) (4 pts) What are the time complexities of Kruskal's algorithm and Prim's algorithm? Which one is asymptotically faster?

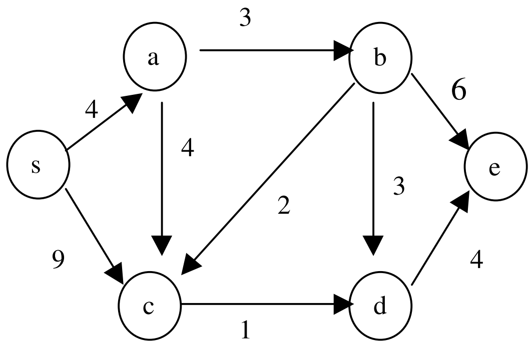
Answer:
Kruskal: O(eloge + nlogn),
Prim: O((e+n)logn),
There are asymptotically the same, because O(eloge) is equal to O(elogn).

(b) (9 pts) The edges of undirected graph *G* are all weighted with the same value. We use breadth-first method to traverse *G* starting from vertex *s*. Can we visit vertex *a* before vertex *b* if the shortest path from *b* to *s* is shorter than the shortest path from *a* to *s*? Justify your answer.

Answer:
No, we can not. Explanation should be clear. Stating clearly the attributes of BFS and shortest path, the reason is obvious.

Problem 3 (15 pts)

Given the following directed graph, use Dijkstra's algorithm to find single-source shortest paths starting from *s* to all vertices. Please fill in the *d* value in the table after each iteration.



Iteration	*d*(s)	*d*(a)	*d*(b)	*d*(c)	*d*(d)	*d*(e)	Extract_min
1	0	∞	∞	∞	∞	∞	S
2	**0**	**4**	∞	**9**	∞	∞	**a**
3	**0**	**4**	**7**	**8**	∞	∞	**b**
4	**0**	**4**	**7**	**8**	**10**	**13**	**c**
5	**0**	**4**	**7**	**8**	**9**	**13**	**d**
6	**0**	**4**	**7**	**8**	**9**	**13**	**e**

Answer: fill in already, the problem is similar with assignment 3 problem 1 this semester.

Problem 4 (15 pts)

A farmer wants to sell his farm produce in market. There are four bags of farm produce he can choose to carry to the market:

- a bag of rice, weight 50 kg, value 40$;
- a bag of corns, weight 40 kg, value 30$;
- a bag of peanuts, weight 30kg, value 50$;
- a bag of apples, weight 30kg, value 60$;

Suppose the farmer can carry 100kg product at most.

(a) (8 pts) If the farmer can carry a whole bag of some farm produce or nothing, what is the maximum value he can carry to the market? Use 0-1 knapsack problem to solve it, please show each step clearly.

Answer: Use the method of assignment 3 problem 3 to solve it.

(b) (7 pts) If the farmer can carry part of some particular farm produce, what is the maximum value he can carry to the market? Use fractional knapsack problem to solve it, please show each step clearly.

Answer: Need a greedy algorithm of "fractional knapsack problem", not covered this semester.

Problem 5 (15 pts)

Suppose we want to send a sequence of digits through a network. There are totally 1000 digits, the time of appearance of each digit are:

digit	0	1	2	3	4	5	6	7	8	9
time	100	400	100	50	50	50	200	10	20	20

(a) (3 pts) If we code every digit with the same bit length, how many bits must we use to send the 1000 digits?
Answer: each digit contains 4 bits, totally 4000 bits.

(b) (12 pts) If we use Huffman coding for the above 10 digits, what is the total bit length for the 1000 digits? Please show the Huffman tree, you need not to write the process of constructing the Huffman tree.
Answer: Use the method of assignment 4 problem 1.

Problem 6 (12 pts)

(a) (2 pts) Pick up all the NP-complete problems from the following problems:
SAT, 2SAT, 3SAT and 4SAT.
Answer: SAT, 3SAT, 4SAT.

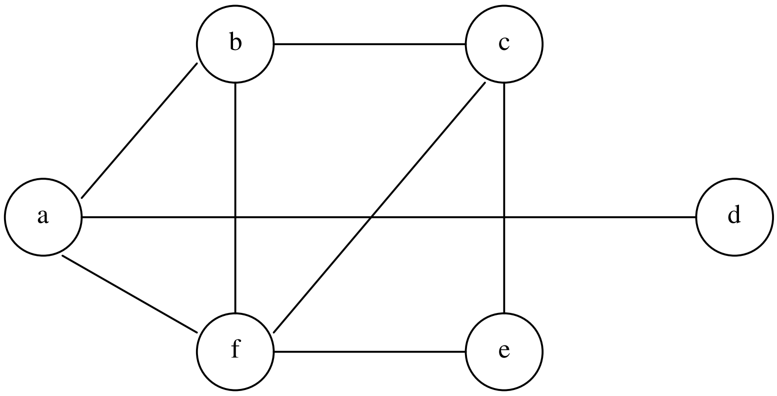
(b) (10 pts) Are the following statements correct? You need not to give the reasons.
1. If a decision problem is in class P, then it is polynomial reducible to SAT.
2. If a decision problem is in class NP, then it is polynomial reducible to SAT.
3. If a decision problem is in class NP, then it must be in class P.
4. If we find a polynomial time algorithm to solve some decision problem, then it must not be NP-complete.
5. If a decision problem is NP-complete, then it is polynomial reducible to the CLIQUE problem.

Answer: T, T, F, F, T

Problem 7 (20 pts)

An independent set of a graph $G=(V,E)$ is a subset $V' \subseteq V$ of vertices such that each edge in E is incident to at most one vertex in V'. The independent-set problem is to find a maximum-size independent set of G.

(a) (4 pts) Find all the possible independent sets for the following graph.



(b) (16 pts) Formulate a related decision problem for the independent-set problem, and prove that it is NP-complete (Hint: Reduce from the CLIQUE problem).

Answer: It is just problem 3 of assignment 4.