# MATH 244 (L2) Applied Statistics

## Mid-Term Examination October 20, 2000

Name _____ Student ID _____ Tutorial section ____

**Time allowed : 1 hour**
**Answer all questions.**

1. (10%) Circle the appropriate answers for the following true or false questions.
**One mark will be given for each correct answer. Half mark will be deducted for each incorrect answer.**

(a) Let $A$ be an event with $0 < \Pr(A) < 1$. It is impossible that the event $A$ and $\overline{A}$ are independent . **T** / F

(b) If $A$ and $B$ are independent events, $\overline{A}$ and $\overline{B}$ are independent too. **T** / F

(c) A group of events are called to be mutually exclusive if any two of them are independent to each other. T / **F**

(d) For any symmetric dataset, sample mean = sample median = mode. T / **F**

(e) If $A$ is an event with $\Pr(A) = 1$, then $A$ must be equal to the sample space. T / **F**

(f) Sample mean is always the best measure of location since it uses all the data. T / **F**

(g) If the distribution of a set of data is skewed to the right, then the median should be closer to the upper quartile than to the lower quartile. T / **F**

(h) A stem-and-leaf plot is more informative than a histogram. **T** / F

(i) If the moment generating function of a random variable $X$ is **T** / F

$$M_X(t) = p/\left[1 - e^t(1-p)\right] \ , \ t < -\ln(1-p) \ ,$$

then $X$ will have the memoryless property.

(j) Histogram is not a suitable form of representation of frequency distribution table when the class-intervals have unequal widths. T / **F**

2. (10%) Suppose John try to answer all the ten questions in problem 1 by pure guess. Let $X$ be the number of correct answers and $Y$ be the marks he can obtain in problem 1.

   (a) What is the distribution of $X$ ? Write down the corresponding assumptions.
   (b) Find the expected value and variance of $X$ .
   (c) Express $Y$ in terms of $X$.
   (d) Use the result in (c), or otherwise, to find the expected value and variance of $Y$.
   (e) What is the probability that John will get negative marks from problem 1?
   (f) What is the probability that John will give the **same** set of answers as you in problem 1?

   **Solution :**

   (a) $X \sim b(10,0.5)$
   Assumptions : For each question, John will answer true with probability 0.5. His choices on the answers of all questions are independent.

   (b) $E(X) = np = (10)(0.5) = 5$
   $Var(X) = np(1-p) = (10)(0.5)(0.5) = 2.5$

   (c) $Y = X + (10 - X)\left(-\frac{1}{2}\right)$ (1 mark for correct answer, -0.5 mark for incorrect answer)
   $= \frac{3}{2}X - 5$

   (d) $E(Y) = \frac{3}{2}E(X) - 5 = \frac{3}{2}(5) - 5 = 2.5$
   $Var(Y) = \left(\frac{3}{2}\right)^2 Var(X) = \frac{45}{8} = 5.625$

   (e) From (c), $Y < 0 \Leftrightarrow \frac{3}{2}X - 5 < 0 \Leftrightarrow X < \frac{10}{3} \Leftrightarrow X \le 3$
   Hence $\Pr(Y < 0) = \Pr(X \le 3)$
   $= \binom{10}{0}(0.5)^0(0.5)^{10} + \binom{10}{1}(0.5)^1(0.5)^9 + \binom{10}{2}(0.5)^2(0.5)^8 + \binom{10}{3}(0.5)^3(0.5)^7$
   $= 0.171875$

   (f) Probability that John will give the same set of answers as me in problem 1
   $= (0.5)^{10} = 9.7656 \times 10^{-4}$
   (If you didn't answer all questions in problem 1, the probability should be zero.)

3. (10%) Three friends A, B and C share a flat. From past experience, there is an average of one incoming telephone call per hour in the evening. The probability of an incoming telephone call in the evening being for A, B, or C is 0.2, 0.3 and 0.5 respectively. The probability that A is at home when it is received is 0.75. The corresponding probabilities for B and C are 0.5 and 0.8 respectively. Assume that the presence of a person is independent to the presences of the others, and is also independent of the telephone calls..

   (a) What is the probability that there is exactly one incoming telephone call from 8:00pm to 11:00pm ?
   (b) Suppose there is one incoming telephone call in the evening, find the probability that
      (i) no-one is at home to answer this telephone call,
      (ii) the call is for A and A is at home,
      (iii) the person to whom the call is made is at home,
      (iv) the call is for A, given that the person to whom the call is made is at home.

**Solution :**

(a) Let $X$ be the number of telephone calls from 8:00pm to 11:00pm.

$$X \sim \wp(3) \quad \text{(average 3 telephone calls in three hours)}$$

$$\Pr(X=1)=\frac{e^{-3}3^{1}}{1!}=0.1494$$

(b) (i) $\Pr(\text{no one is at home to answer the call})=(1-0.75)(1-0.5)(1-0.8)=0.025$

(ii) $\Pr(\text{the call is for A and A is at home})=(0.2)(0.75)=0.15$

(iii)
$$\begin{aligned}&\Pr(\text{the person to whom the call is made is at home})\\ &=\Pr(\text{the call is for A and A is at home})+\Pr(\text{the call is for B and B is at home})\\ &\quad+\Pr(\text{the call is for C and C is at home})\\ &=(0.2)(0.75)+(0.3)(0.5)+(0.5)(0.8)=0.7\end{aligned}$$

(iv)
$$\begin{aligned}&\Pr(\text{the call is for A}\mid\text{the person to whom the call is made is at home})\\ &=\frac{\Pr(\text{the call is for A}\cap\text{the person to whom the call is made is at home})}{\Pr(\text{the person to whom the call is made is at home})}\\ &=\frac{\Pr(\text{the call is for A and A is at home})}{\Pr(\text{the person to whom the call is made is at home})}\\ &=\frac{(0.2)(0.75)}{0.7}=0.2143\end{aligned}$$

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