1.

(a) Positively skewed.

(b)

CPU time (s)	Frequency	Cumulative Frequency
0.2 – 1.0	960	960
1.0 – 1.8	2880	3840
1.8 – 2.8	3000	6840
2.8 – 3.8	1600	8440
3.8 – 5.6	1080	9520
5.6 – 8.0	480	10000
Total	10000	

(c) Min = 0.2 , Max = 8.0

For Q_L, $p = 0.25$, $np = 10000 \times 0.25 = 2500$,

$$Q_L = 1.0 + \frac{(1.8 - 1.0)(2500 - 960)}{2880} = 1.43$$

For median, $p = 0.5$, $np = 10000 \times 0.5 = 5000$,

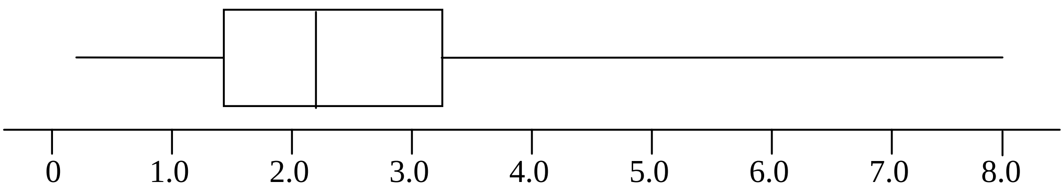
$$meidan = 1.8 + \frac{(2.8 - 1.8)(5000 - 3840)}{3000} = 2.19$$

For Q_U, $p = 0.75$, $np = 10000 \times 0.75 = 7500$,

$$Q_U = 2.8 + \frac{(3.8 - 2.8)(7500 - 6840)}{1600} = 3.21$$

Hence the five number summary is : 0.2, 1.43, 2.19, 3.21, 8.0 .

(d) Boxplot for the data :



2. Joint and marginal distributions of *X* and *Y*:

	Y			
X	**0**	**1**	**2**	**Marginal**
1	0	0.05	0.1	0.15
2	0.15	0.1	0.15	0.4
3	0.3	0.15	0	0.45
Marginal	0.45	0.3	0.25	1

(a) $\Pr(X = 1)\Pr(Y = 0) = 0.15 \times 0.45 = 0.0675 \neq 0 = \Pr(X = 1, Y = 0)$

Hence *X* and *Y* are not independent.

2. (b) $E(X) = 1\times 0.15 + 2\times 0.4 + 3\times 0.45 = 2.3$

$E(X^2) = 1^2\times 0.15 + 2^2\times 0.4 + 3^2\times 0.45 = 5.8$

$Var(X) = 5.8 - 2.3^2 = 0.51$

$E(Y) = 0\times 0.45 + 1\times 0.3 + 2\times 0.25 = 0.8$

$E(Y^2) = 0^2\times 0.45 + 1^2\times 0.3 + 2^2\times 0.25 = 1.3$

$Var(Y) = 1.3 - 0.8^2 = 0.66$

$E(XY) = (1)(0)(0) + (1)(1)(0.05) + \cdots + (3)(2)(0) = 1.5$

$Cov(X,Y) = 1.5 - 2.3\times 0.8 = -0.34$

$$\rho = \frac{-0.34}{\sqrt{0.51\times 0.66}} = -0.5860$$

(c) $\Pr(\text{at least 2 machines available}) = \Pr(X \geq 2) = 0.4 + 0.45 = 0.85$

$\Pr(\text{exactly one operation} \cap \text{at least 2 machines available}) = \Pr(Y = 1, X \geq 2)$

$= 0.1 + 0.15 = 0.25$

$\Pr(\text{exactly one operation} \mid \text{at least 2 machines available})$

$$= \Pr(Y = 1 \mid X \geq 2) = \frac{\Pr(Y = 1, X \geq 2)}{\Pr(X \geq 2)} = \frac{0.25}{0.85} = 0.2941$$

3. Let X be the number of jobs submitted to the system from 3:00pm to 3:30pm. Then $X \sim \wp(6)$.

$$\Pr(X \geq 3) = 1 - e^{-6}\left(1 + 6 + \frac{(6)^2}{2!}\right) = 0.9380$$

4. $\mu = 200$, $\sigma = 55$

Let X_i, $i = 1,2,\ldots,45$ be the weight of the 45 boxes. Then by normal approximation,

$$\overline{X} \,\dot{\sim}\, N\left(200, \frac{55^2}{45}\right).$$

$$\Pr(\text{all boxes can be loaded}) = \Pr\left(\sum_{i=1}^{45} X_i \leq 10000\right) = \Pr\left(\overline{X} \leq \frac{10000}{45}\right)$$

$$\approx \Phi\left(\frac{10000/45 - 200}{55/\sqrt{45}}\right) = \Phi(2.71) = 0.9966$$

5. (a) $\Pr(\text{lucky ticket}) = 0.001 + 0.002 + 0.004 + 0.01 + 0.03 = 0.047$

(b) Let W be the prize of one ticket. Then $X = W - 10$.

$E(W) = 2000\times 0.001 + 1000\times 0.002 + 250\times 0.004 + 100\times 0.01 + 50\times 0.03 = 7.5$

$E(W^2) = 2000^2\times 0.001 + 1000^2\times 0.002 + 250^2\times 0.004 + 100^2\times 0.01 + 50^2\times 0.03 = 6425$

$Var(W) = 6425 - 7.5^2 = 6368.75$

$E(X) = E(W) - 10 = -2.5$, $Var(X) = Var(W) = 6368.75$

(c) Let Y be the number of lucky tickets in 100 ticket purchased. Then $Y \sim b(100, 0.047)$.

$\Pr(Y < 5) = (0.953)^{100} + {}_{100}C_1(0.953)^{99}(0.047) + \cdots + {}_{100}C_4(0.953)^{96}(0.047)^4 = 0.4915$

(d) Let X_i, $i = 1,2,...,100$ be the net gains by each of the 100 tickets. Then by normal approximation,

$$\overline{X} \dot{\sim} N\left(-2.5, \frac{6368.75}{100}\right).$$

Hence

$$\Pr\left(\sum_{i=1}^{100} X_i > -100\right) = \Pr\left(\overline{X} > -1\right) \approx 1 - \Phi\left(\frac{-1-(-2.5)}{\sqrt{6368.75/100}}\right)$$
$$= 1 - \Phi(0.188) = 1 - 0.5745 = 0.4255$$

(e) The tickets purchased were assumed to be independent to each other.