1.

(a) Positively skewed.

(b) Min = 0 , Max = 650

For Q_L , $p = 0.25$, $np = 185 \times 0.25 = 46.25$,

$$Q_L = 100 + \frac{100(46.25 - 30)}{40} = 140.625$$

For median, $p = 0.5$, $np = 185 \times 0.5 = 92.5$,

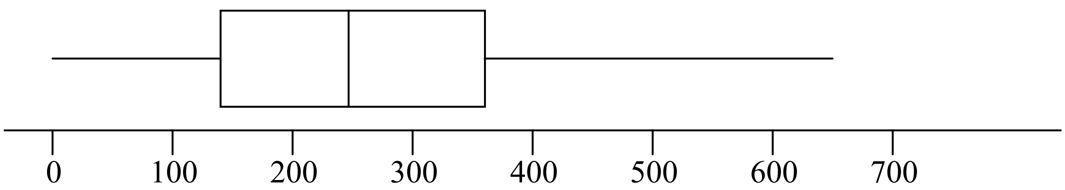
$$meidan = 200 + \frac{100(92.5 - 70)}{50} = 245$$

For Q_U , $p = 0.75$, $np = 185 \times 0.75 = 138.75$,

$$Q_U = 300 + \frac{150(138.75 - 120)}{45} = 362.5$$

Hence the five number summary is : 0, 140.625, 245, 362.5, 650 .

(c) Boxplot for the data :



(d) Total amount spent by customers who spent more than USD200

$\approx 250 \times 50 + 375 \times 45 + 550 \times 20$

$= 40375$

Hence $\Pr(\text{you will will the prize}) = \dfrac{452}{\text{total amount spent customers who spent more than 200}}$.

$$\approx \frac{452}{40375} = 0.01120$$

2. Let A, B, C, D be the event that each hunter can hit the lion respectively.

(a) $\Pr(\text{lion will be hit}) = 1 - \Pr(\overline{A} \cap \overline{B} \cap \overline{C} \cap \overline{D})$

$= 1 - \Pr(\overline{A})\Pr(\overline{B})\Pr(\overline{C})\Pr(\vec{D})$

$= 1 - (0.8)(0.6)(0.7)(0.9) = 0.6976$

(b) $\Pr(X = 0) = \Pr(\overline{A} \cap \overline{B} \cap \overline{C} \cap \overline{D}) = 0.3024$

$\Pr(X = 4) = \Pr(A \cap B \cap C \cap D) = (0.2)(0.4)(0.3)(0.1) = 0.0024$

$\Pr(X = 3) = \Pr(\overline{A} \cap B \cap C \cap D) + \Pr(A \cap \overline{B} \cap C \cap D) + \Pr(A \cap B \cap \overline{C} \cap D)$

$+ \Pr(A \cap B \cap C \cap \overline{D})$

$= (0.8)(0.4)(0.3)(0.1) + (0.2)(0.6)(0.3)(0.1) + (0.2)(0.4)(0.7)(0.1) + (0.2)(0.4)(0.3)(0.9)$

$= 0.0404$

$$\Pr(X=1)=\Pr(A\cap\overline{B}\cap\overline{C}\cap\overline{D})+\Pr(\overline{A}\cap B\cap\overline{C}\cap\overline{D})+\Pr(\overline{A}\cap\overline{B}\cap C\cap\overline{D})$$
$$+\Pr(\overline{A}\cap\overline{B}\cap\overline{C}\cap D)$$
$$=(0.2)(0.6)(0.7)(0.9)+(0.8)(0.4)(0.7)(0.9)+(0.8)(0.6)(0.3)(0.9)+(0.8)(0.6)(0.7)(0.1)$$
$$=0.4404$$
$$\Pr(X=2)=1-0.3024-0.0024-0.0404-0.4404=0.2144$$

Hence the pdf of X is :

$$\Pr(X=x)=\begin{cases}0.3024 & x=0\\ 0.4404 & x=1\\ 0.2144 & x=2\\ 0.0404 & x=3\\ 0.0024 & x=4\end{cases}$$

(c) $E(X)=(0)(0.3024)+(1)(0.4404)+(2)(0.2144)+(3)(0.0404)+(4)(0.0024)=1$

$E(X^2)=(0)^2(0.3024)+(1)^2(0.4404)+(2)^2(0.2144)+(3)^2(0.0404)+(4)^2(0.0024)=1.7$

$Var(X)=E(X^2)-[E(X)]^2=1.7-1^2=0.7$

$\sigma=\sqrt{Var(X)}=\sqrt{0.7}=0.8367$

(d) From (b), $\Pr(X=1)=0.4404$.

$$\Pr(\text{A kill the lion}\mid X=1)=\frac{\Pr(\text{A kill the lion}\cap X=1)}{\Pr(X=1)}$$
$$=\frac{\Pr(A\cap\overline{B}\cap\overline{C}\cap\overline{D})}{\Pr(X=1)}$$
$$=\frac{(0.2)(0.6)(0.7)(0.9)}{0.4404}=0.1717$$

$$\Pr(\text{B kill the lion}\mid X=1)=\frac{(0.8)(0.4)(0.7)(0.9)}{0.4404}=0.4578$$

$$\Pr(\text{C kill the lion}\mid X=1)=\frac{(0.8)(0.6)(0.3)(0.9)}{0.4404}=0.2943$$

$$\Pr(\text{D kill the lion}\mid X=1)=\frac{(0.8)(0.6)(0.7)(0.1)}{0.4404}=0.0763$$

3.

(a) Let X be the number of customers arrived from 9:00am to 9:30am. Then $X\sim\wp(4.5)$.

$$\Pr(X>5)=1-e^{-4.5}\left(1+4.5+\frac{(4.5)^2}{2!}+\frac{(4.5)^3}{3!}+\frac{(4.5)^4}{4!}+\frac{(4.5)^5}{5!}\right)=0.2971$$

(b) $\Pr(X>5\mid X\geq 1)=\dfrac{\Pr(X>5)}{\Pr(X\geq 1)}=\dfrac{0.2971}{1-e^{-4.5}}=0.3004$

(c) From the independent assumption of Poisson process, occurrence of events in nonoverlapping time intervals are independent. Hence

$$\Pr(X > 5 \mid \text{first customer arrive at } 9:10\text{am}) = \Pr(Y > 4)$$

where $Y \sim \wp(3)$ is the number of customers from 9:10am to 9:30am

$$= 1 - e^{-3}\left(1 + 3 + \frac{3^2}{2!} + \frac{3^3}{3!} + \frac{3^4}{4!}\right)$$

$$= 0.1847$$

4.

(a) $X \sim b(9, 0.4)$

(b) $\Pr(X \geq 5) = \binom{9}{5}(0.4)^5(0.6)^4 + \cdots + \binom{9}{9}(0.4)^9(0.6)^0 = 0.2666$

(c) $Y = 5X - 3(9 - X) = 8X - 27$

$E(Y) = 8E(X) - 27 = 8(9)(0.4) - 27 = 1.8$

$Var(Y) = 64Var(X) = 64(9)(0.4)(0.6) = 138.24$

Since $E(Y) \neq 0$, it is not a fair game.

(d) $\Pr(Y < 0) = \Pr(8X - 27 < 0) = \Pr(X < 3.375)$

$$= \binom{9}{0}(0.4)^0(0.6)^9 + \cdots + \binom{9}{3}(0.4)^3(0.6)^6 = 0.4826$$