MATH 244 (L1) Applied Statistics

Quiz 2

Name ___________________ Student ID ______________ Tutorial section ____

Time allowed : 45 minutes

1. (*6 marks*) Customers arrive at a shop on the average of one per minute in accordance with a Poisson process.

(a) Find the probability that there will be at most 3 customers from 10:00am to 10:05am.

Time unit = minute $\lambda = 1$

$N(5) \sim \wp(5)$

$\Pr(N(5) \le 3) = e^{-5}\left\{1 + \frac{5}{1!} + \frac{5^2}{2!}\right\} = 0.1247$

(b) Find the probability that there will be less than 30 customers within 37.2 minutes.

Let T = waiting time (in minutes) until the 30th customer. Then $T \sim \Gamma(30,1)$, or $2T \sim \Gamma\left(30, \frac{1}{2}\right) \equiv \chi^2_{60}$. Hence

$$\begin{aligned}\Pr(N(37.2) < 30) &= \Pr(T > 37.2)\\ &= \Pr(2T > 74.4)\\ &= \Pr(\chi^2_{60} > 74.4) = 1 - 0.90 = 0.10\end{aligned}$$

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2. (*10 marks*) A recruitment agency evaluates, for each applicant, an aptitude rating based on many factors and tests. The result of such a rating is a continuous quantity that is normally distributed with mean 80 and standard deviation 18. An employer considers applicants with a rating of 100 or more as suitable, and those with a rating of 120 or more as outstanding.

(a) What proportion of applicants are suitable?

$X \sim N\left(80, 18^2\right)$

% of suitable applicant = $\Pr(X \geq 100)$

$$= 1 - \Phi\left(\frac{100-80}{18}\right) = 1 - \Phi(1.11) = 1 - 0.8665 = 13.35\%$$

(b) What proportion of **suitable** applicants are outstanding?

% of outstanding applicant = $\Pr(X \geq 120)$

$$= 1 - \Phi\left(\frac{120-80}{18}\right) = 1 - \Phi(2.22) = 1 - 0.9868 = 1.32\%$$

% of suitable applicants who are outstanding

$$= \Pr(X \geq 120 \mid X \geq 100) = \frac{\Pr(X \geq 120)}{\Pr(X \geq 100)} = \frac{0.0132}{0.1335} = 9.89\%$$

(c) Suppose forty applicants take the test independently. What are the mean and standard deviation of the number of applicants that will be considered as suitable?

$Y = \text{number of suitable applicants}$ $\qquad Y \sim b(40, 0.1335)$

$E(Y) = (40)(0.1335) = 5.34$

$Var(Y) = (40)(0.1335)(1 - 0.1335) = 4.6271$

$\sqrt{Var(Y)} = 2.1511$

3. (*9 marks*) One investment consultant believes that the probability distribution of returns (in percent per year) on one particular international fund is as given below:

Return rate	-2%	-1%	0%	3%	6%	9%	12%
Prob.	0.05	0.13	0.17	0.30	0.15	0.15	0.05

(a) Find the mean and standard deviation of the return rate of this fund.

$$E(R) = (-2)(0.05) + \cdots + (12)(0.05) = 3.52\%$$
$$E(R^2) = (-2)^2(0.05) + \cdots + (12)^2(0.05) = 27.78\,(\%^2)$$
$$Var(R) = E(R^2) - E(R)^2 = 15.3896\,(\%^2)$$
$$\sqrt{Var(R)} = 3.92\%$$

(b) Bob is going to invest part of his money on this fund and keep the rest in his saving account. Assume that saving interest will be paid yearly with interest rate 2% per year. What proportion should he invest on the fund so that he can expect to have 3% overall returns after one year?

Let p be the required proportion. Then return after one year = $pR + 0.02(1-p)$. Hence

$$\begin{aligned} 0.03 &= E(pR + 0.02(1-p)) = pE(R) + 0.02(1-p) \\ &= 0.0352p + 0.02(1-p) = 0.0152p + 0.02 \end{aligned}$$
$$\Rightarrow \qquad p = \frac{0.03 - 0.02}{0.0152} = 65.79\%$$

(c) What is the standard deviation of the overall return rate of Bob's portfolio in part (b)?

Standard deviation of overall return rate = $p\sqrt{Var(R)}$

$$= (0.6579)(0.0392) = 2.58\%$$

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