MATH 244 (L2)   Applied Statistics

Quiz 3

Name ____________________   Student ID ______________   Tutorial section ____

Time allowed : 45 minutes

1. (*8 marks*) A poll selects a random sample of 100 Hong Kong citizens and asks each person selected if they agree with the government's proposal dealing with the economic crisis. Suppose 99 of the individuals agree and the rest do not.

(a) Construct a 95% confidence interval for the population proportion of Hong Kong citizens who agree with the government's proposal.

$$\hat{\pi} \pm Z_{0.025}\sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{n}} = 0.99 \pm (1.96)\sqrt{\frac{(0.99)(0.01)}{100}} = 0.99 \pm 0.0195 = [97.05\%\,,100.95\%]$$

Since population proportion should be less than one, a more reasonable confidence interval should be $[97.05\%\,,100\%)$.

(b) If you are asked to conduct the survey again so that you can produce a 95% confidence interval with margin of error at most 0.02, how many Hong Kong citizens should you sample?

$$n = \frac{Z_{0.025}^2\hat{\pi}(1-\hat{\pi})}{D^2} = \frac{(1.96)^2(0.99)(0.01)}{(0.02)^2} = 95.08 \approx 96$$

(c) Write down the assumption(s) you had made in the calculations in (a) and (b).

Assumptions : (i) The population size is very large.
(ii) The sample is randomly chosen.

P. T. O

2. (*8 marks*) An automobile manufacturer wants to make sure a car's brakes are free from defects before installing them in a new car.

(a) State the appropriate null and alternative hypothesis about the brakes. (Take the perspective that the manufacturer's main concern is to avoid installing defective brakes.)

H_0 : The brakes are defective.

H_1 : The brakes are non-defective.

(b) In terms of this setting, describe what will be the consequences of Type I and Type II errors.

Type I error : Installing defective brakes which may result in traffic accident.

Type II error : Abandon non-defective brakes which may increase the cost on manufacturing new cars.

(c) In an unrelated setting, a film developer, wishing to determine whether or not the colours setting for his equipment are correct, adopts the following hypotheses:

H_0 : The settings are incorrect.

H_1 : The settings are correct.

In which of these two settings, setting (a) or setting (c), would you choose a smaller value for α (type I error probability)? Explain briefly.

If the film developer make a type I error in setting (c), he would just ruin some photos due to the use of incorrect colour settings. The type I error in setting (a), however, may result in fatal traffic accident which is much more serious. Therefore a smaller value for α should be used in setting (a).

3. (*9 marks*) Two alloys, A and B, are used in the manufacture of steel bars. Suppose a steel producer wants to compare the two alloys on the basis of average load capacity, where the load capacity of a steel bar is defined as the maximum load (weight in tons) it can support without breaking. Steel bars containing alloy A and steel bars containing alloy B were randomly selected and tested for load capacity. The results are summarized in the accompany table.

Alloy A	Alloy B
$m = 17$	$n = 11$
$\bar{X} = 43.7$	$\bar{Y} = 48.5$
$S_x^2 = 24.4$	$S_y^2 = 19.9$

(a) Find a 95% confidence interval for the difference between the true average load capacities for the two alloys. State the assumption(s) you had made.

Assumptions : (i) Population load capacities is normal.
(ii) The two samples are independent.
(iii) The population variances of load capacities are the same.

$$S_{pool}^2 = \frac{16 \times 24.4 + 10 \times 19.9}{16 + 10} = 22.67$$

95% C.I. for $\mu_A - \mu_B$:

$$\left(\bar{X} - \bar{Y}\right) \pm t_{26,0.025} S_{pool} \sqrt{\frac{1}{17} + \frac{1}{11}} = (43.7 - 48.5) \pm 2.056 \sqrt{(22.67)\left(\frac{1}{17} + \frac{1}{11}\right)}$$
$$= -4.8 \pm 3.788 = [-8.588\,,\,-1.012]$$

(b) What can be concluded from the confidence interval obtained in (a)?

Since all the values of the interval estimate are negative, we have high confidence that $\mu_A - \mu_B$ is negative, i.e. the true average load capacity of alloy A is less than that of alloy B by at least 1.012 tons.

(c) How many steel bars of alloy B should be sampled in order to estimate the true average load capacity for alloy B to within 1.5 tons with 99% confidence?

$$n = \frac{Z_{0.005}^2 \hat{\sigma}_y^2}{D^2} = \frac{(2.576)^2 (19.9)}{(1.5)^2} = 58.69 \approx 59$$

<E N D >