MATH 244 (L2) Applied Statistics

Quiz 1

Name ____________________ Student ID _______________ Tutorial section ____

Time allowed : 45 minutes

1. (*9 marks*) A machine is able to operate effectively as long as both of its two components are functioning. Within a certain period, the chance that the first component will fail is 0.4; the chance that the second component will fail is 0.7; and the chance that the second component will fail while the first still function is 0.42.

(a) Are the failures of these two components independent events? Why?

Let A be the event that the first component fail, B be the event that the second component fail.

$\Pr(\text{both fail}) = \Pr(A \cap B) = \Pr(B) - \Pr(\overline{A} \cap B) = 0.7 - 0.42 = 0.28$
$\Pr(A)\Pr(B) = 0.4 \times 0.7 = 0.28 = \Pr(A \cap B)$
They are independent.

(b) Are the failures of these two components disjoint events? Why?

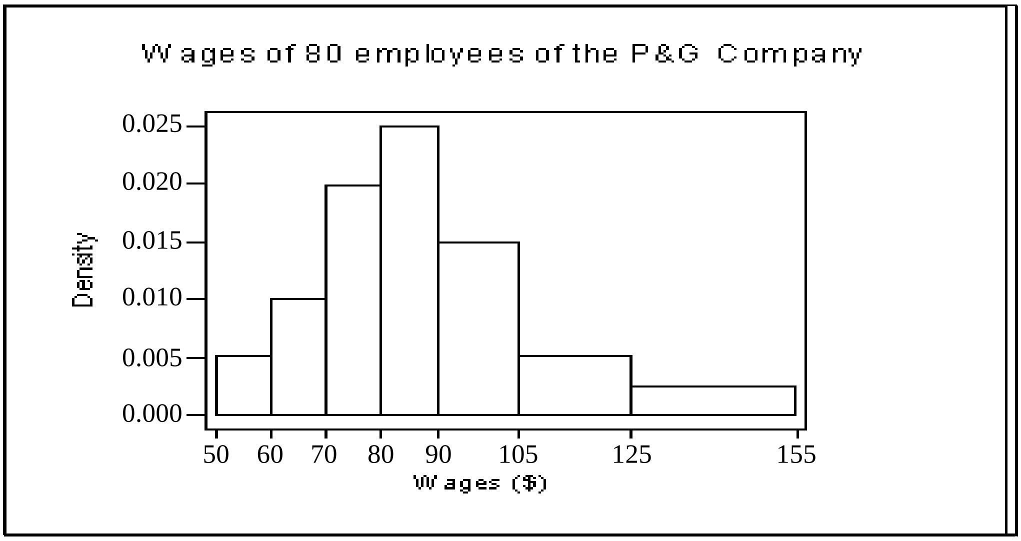
If they are disjoint, $\Pr(A \cap B)$ will be zero. Now $\Pr(A \cap B) = 0.28$. Therefore they are not disjoint.

(c) If you found that the machine stop functioning, what is the probability that both components had failed to operate?

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B) = 0.4 + 0.7 - 0.28 = 0.82$$

$$\Pr(A \cap B \mid A \cup B) = \frac{\Pr((A \cap B) \cap (A \cup B))}{\Pr(A \cup B)} = \frac{\Pr(A \cap B)}{\Pr(A \cup B)} = \frac{0.28}{0.82} = \frac{14}{41} = 0.3415$$

2. (*8 marks*) The following histogram shows the distribution of the weekly wages in dollars of 80 employees at the P&R Company.



(a) Complete the following frequency distribution table of the same set of data.

Wages ($)	**Frequency**	**Cumulative Frequency**
50 – 60	4	4
60 – 70	8	12
70 – 80	16	28
80 – 90	20	48
90 – 105	18	66
105 – 125	8	74
125 - 155	6	80
Total	80	

(b) Find the five number summary and hence sketch the boxplot for the data.

Min = 50 , Max = 155

For Q_L, $p = 0.25$, $np = 80 \times 0.25 = 20$

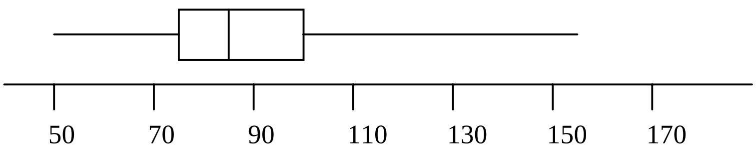
$$Q_L = 70 + \frac{(10)(20-12)}{16} = 75$$

For median, $p = 0.5$, $np = 80 \times 0.5 = 40$

$$median = 80 + \frac{(10)(40-28)}{20} = 86$$

For Q_U, $p = 0.75$, $np = 80 \times 0.75 = 60$

$$Q_U = 90 + \frac{(15)(60-48)}{18} = 100$$



3. (*8 marks*) The Department of Wildlife needs to estimate the number of deer within a wildlife preserve. Deer do not generally cooperate by standing still to be counted, so a more subtle technique has been developed. Suppose that the preserve is known to contain either 1, 2, 3, 4, or 5 deer with equal probability. We choose a deer at random, tag it, and let it go. The next day we observe a deer from this preserve and note whether it is tagged or not.

(a) What is the probability that the observed deer is **not** tagged?

$$\Pr(1\ deer) = \Pr(2\ deer) = \Pr(3\ deer) = \Pr(4\ deer) = \Pr(5\ deer) = \frac{1}{5}$$

$$\Pr(not\ tagged \mid 1\ deer) = \frac{0}{1} = 0 \quad , \quad \Pr(not\ tagged \mid 2\ deer) = \frac{1}{2}$$

$$\Pr(not\ tagged \mid 3\ deer) = \frac{2}{3} \quad , \quad \Pr(not\ tagged \mid 4\ deer) = \frac{3}{4}$$

$$\Pr(not\ tagged \mid 5\ deer) = \frac{4}{5}$$

$$\Pr(not\ agged) = \Pr(not\ tagged \mid 1\ deer)\Pr(1\ deer) + \cdots + \Pr(not\ tagged \mid 5\ deer)\Pr(5\ deer)$$

$$= 0 \times \frac{1}{5} + \frac{1}{2} \times \frac{1}{5} + \frac{2}{3} \times \frac{1}{5} + \frac{3}{4} \times \frac{1}{5} + \frac{4}{5} \times \frac{1}{5} = \frac{163}{300} = 0.5433$$

(b) Suppose the observed deer is tagged. What are the posterior probabilities that the preserve contains 1, 2, 3, 4, 5 deer, respectively?

$$\Pr(tagged) = 1 - \Pr(not\ tagged) = \frac{137}{300} = 0.4567$$

$$\Pr(1\ deer \mid tagged) = \frac{\Pr(tagged \mid 1\ deer)\Pr(1\ deer)}{\Pr(tagged)} = \frac{1}{1} \times \frac{1}{5} \Big/ \frac{137}{300} = \frac{60}{137} = 0.4380$$

$$\Pr(2\ deer \mid tagged) = \frac{\Pr(tagged \mid 2\ deer)\Pr(2\ deer)}{\Pr(tagged)}$$

$$= \frac{1}{2} \times \frac{1}{5} \Big/ \frac{137}{300} = \frac{30}{137} = 0.2190$$

$$\Pr(3\ deer \mid tagged) = \frac{1}{3} \times \frac{1}{5} \Big/ \frac{137}{300} = \frac{20}{137} = 0.1460$$

$$\Pr(4\ deer \mid tagged) = \frac{1}{4} \times \frac{1}{5} \Big/ \frac{163}{300} = \frac{15}{137} = 0.1095$$

$$\Pr(5\ deer \mid tagged) = \frac{1}{5} \times \frac{1}{5} \Big/ \frac{137}{300} = \frac{12}{137} = 0.0876$$

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