MATH 244 Applied Statistics

Quiz 2

Name ___________________ Student ID ______________ Tutorial section ____

Time allowed : 45 minutes

1. An event W occurs with probability 0.4. If A occurs, then the probability of W is 0.6; if A does not occur but B occurs, the probability of W is 0.1, while if neither A nor B occur, the probability of W is 0.5. Finally, if A does not occur, the probability of B is 0.3. Find Pr(A).

Solution

$\Pr(W) = 0.4$, $\Pr(W \mid A) = 0.6$, $\Pr(W \mid \overline{A} \cap B) = 0.1$
$\Pr(W \mid \overline{A} \cap \overline{B}) = 0.5$, $\Pr(B \mid \overline{A}) = 0.3$

Let $p = \Pr(A)$, then

$$\Pr(\overline{A}) = 1 - p$$

$$\Pr(\overline{A} \cap B) = \Pr(B \mid \overline{A})\Pr(\overline{A}) = 0.3(1 - p)$$

$$\Pr(\overline{B} \mid \overline{A}) = 1 - \Pr(B \mid \overline{A}) = 1 - 0.3 = 0.7$$

$$\Pr(\overline{A} \cap \overline{B}) = \Pr(\overline{B} \mid \overline{A})\Pr(\overline{A}) = 0.7(1 - p)$$

$$\Pr(W \cap \overline{A} \cap B) = \Pr(W \mid \overline{A} \cap B)\Pr(\overline{A} \cap B) = 0.03(1 - p)$$

$$\Pr(W \cap \overline{A} \cap \overline{B}) = \Pr(W \mid \overline{A} \cap \overline{B})\Pr(\overline{A} \cap \overline{B}) = 0.35(1 - p)$$

$$\Pr(W \cap \overline{A}) = \Pr(W \cap \overline{A} \cap B) + \Pr(W \cap \overline{A} \cap \overline{B}) = 0.38(1 - p)$$

$$\Pr(W \cap A) = \Pr(W \mid A)\Pr(A) = 0.6p$$

$$\Pr(W) = \Pr(W \cap A) + \Pr(W \cap \overline{A}) = 0.38 - 0.38p + 0.6p = 0.38 + 0.22p$$

Hence

$$0.4 = 0.38 + 0.22p$$

i.e. $$p = \frac{1}{11} = 0.0909$$

P. T. O.

2. Two different suppliers, A and B, provide the manufacturer with the same part. All supplies of this part are kept in a large bin. In the past, 2% of all parts supplied by A and 4% of parts supplied by B have been defective. Moreover, A supplies three times as many parts as B.

(a) Suppose that you reach into the bin and select a part. Find the probability that this part is defective.

(b) Suppose that you reach into the bin, select a part, and find it nondefective. What is the probability that it was supplied by B?

Solution

(a)
$$\begin{aligned}\Pr(\text{defective}) &= \Pr(\text{defective} \mid \text{A})\Pr(A) + \Pr(\text{defective} \mid \text{B})\Pr(B) \\ &= (0.02)(0.75) + (0.04)(0.25) = 0.025\end{aligned}$$

(b)
$$\Pr(B \mid \text{nondefective}) = \frac{\Pr(\text{nondefective} \mid \text{B})\Pr(B)}{\Pr(\text{nondefective})} = \frac{(1-0.04)(0.25)}{1-0.025} = 0.2462$$

3. On the basis of past experience, the buyer for a large sports store estimates that the number of 10-speed bicycles sold next year will be somewhere between 40 and 90 – with the following distribution:

Number of Bicycles Sold (*X*)	**Probability**
40	0.05
50	0.15
60	0.41
70	0.34
80	0.04
90	0.01

(a) What is the mean number sold? What is the standard deviation?
(b) If 60 bicycles are ordered, what is the chance they will all be sold?
(c) Suppose each sold bicycle yields a net profit of 200 dollars and each unsold bicycle yields a net loss of 100 dollars. What is the expected profit the shopper can make in the coming year if he only purchases 60 bicycles?

Solution

(a) $E(X) = (40)(0.05) + (50)(0.15) + \cdots + (90)(0.01) = 62$

$$E(X^2) = (40)^2(0.05) + (50)^2(0.15) + \cdots + (90)^2(0.01) = 3934$$

$$Var(X) = E(X^2) - [E(X)]^2 = 3934 - (62)^2 = 90$$

$$\sigma = \sqrt{Var(X)} = \sqrt{90} = 9.49$$

(b) $\Pr(\text{all 60 bicycles will be sold}) = \Pr(X \geq 60) = 0.41 + 0.34 + 0.04 + 0.01 = 0.8$

(c)

Number of Bicycles Sold (*X*)	**Probability**	**Net Profit**
40	0.05	6000
50	0.15	9000
60	0.41	12000
70	0.34	12000
80	0.04	12000
90	0.01	12000

$$E(\text{Profit}) = (6000)(0.05) + (9000)(0.15) + \cdots + (12000)(0.01) = 11250$$

< E N D >