MATH 244 Applied Statistics

Quiz 4

Name ____________________ Student ID ______________ Tutorial section ____

Time allowed : 45 minutes

1. In a quality control sample of 1,200 randomly selected mobile phones of a certain brand, 52 were found to emit high-level radiation.
   (a) Give a 99% confidence interval for the proportion of all mobile phones of this brand that emit high-level radiation.
   (b) If you are asked to do the survey again and want to have a 99% confidence interval with length less than 0.02, how large your sample should be?

Solution

(a) A 99% confidence interval for the proportion is

$$\hat{p} \pm Z_{0.005}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \frac{52}{1200} \pm (2.576)\sqrt{\frac{(52/1200)(1148/1200)}{1200}}$$

$$= 0.04333 \pm 0.01514 = [0.02819\ ,\ 0.05847]$$

(b) margin of error of the required interval is 0.01

Sample size needed = $\dfrac{(Z_{0.005})^2\,\hat{p}(1-\hat{p})}{D^2} = \dfrac{(2.576)^2(52/1200)(1148/1200)}{(0.01)^2} = 2750.90$

Hence a sample with size 2751 should be needed.

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2. The Choice Magazine had studied the length of life of light bulbs. A random sample of 61 brand X light bulbs yielded a mean of 937.4 hours and variance 784 hour^2. Another sample of 41 brand Y light bulbs yielded a mean of 988.9 hours and variance 427 hour^2. Assume normal populations and independent samples. Let μ_X, μ_Y, σ_x^2, σ_y^2 be the mean lifetimes and variance of lifetimes of brand X light bulbs and brand Y light bulbs respectively.

(a) Find a 90% confidence interval for σ_x^2/σ_y^2. What can be concluded from this interval?

(b) Find a 95% confidence interval for $\mu_X - \mu_Y$. What can be concluded from this interval?

Solution

(a) A 90% confidence interval for σ_x^2/σ_y^2 is given by

$$\left[\frac{1}{F(60,40,0.05)}\frac{S_x^2}{S_y^2}\ ,\ F(40,60,0.05)\frac{S_x^2}{S_y^2}\right]=\left[\frac{1}{1.64}\frac{784}{427}\ ,\ (1.59)\frac{784}{427}\right]=[1.120\ ,\ 2.365]$$

Since all the values in this interval are greater than one, it can be concluded that $\sigma_x^2 > \sigma_y^2$ with high confidence.

(b) From part (a), the variances seem to be unequal. However, since sample sizes are large, the 95% confidence interval for $\mu_X - \mu_Y$ is approximated by

$$\begin{aligned}(\overline{X}-\overline{Y})\pm Z_{0.025}\sqrt{\frac{S_x^2}{61}+\frac{S_y^2}{41}} &= (937.4-988.9)\pm(1.96)\sqrt{\frac{784}{61}+\frac{427}{41}}\\ &= -51.5\pm 9.454\\ &= [-60.954\ ,\ -42.046]\end{aligned}$$

Since all the values in this interval are less than zero, it can be concluded that $\mu_x < \mu_y$ with high confidence, i.e. brand Y light bulbs have longer average lifetime than brand X light bulbs.

3. In a study, the hypothesis $H_0 : \mu \leq 20$ against $H_1 : \mu > 20$ was tested. Based on a random sample with size 100, the null hypothesis was not rejected with significance level $\alpha = 0.05$. A 95% confidence interval for the population mean μ was obtained as [-18.1 , 23.4].

Which of the following statements are incorrect? Explain briefly for each incorrect statement.

(a) "The data provided strong evidence that $\mu \leq 20$."
(b) "The probability that the null hypothesis is correct is 0.95."
(c) "If we draw another random sample with the same size and do the test again, it will be possible to reject the null hypothesis."
(d) "The probability that the population mean is between –18.1 and 23.4 is 0.95."
(e) "If we draw another random sample with the same size and construct the confidence interval again, it will be possible to have an interval which does not contain 20."
(f) "Double the sample size will half the length of the confidence interval obtained."

Solution

(a) Incorrect. Since we didn't control the type II error probability to a small value, we may have great chance to falsely accept the null hypothesis although the result is a non-rejection. The data didn't have enough evidence to reject the null hypothesis. But it doesn't mean that the data provide strong evidence to accept the null hypothesis.

(b) Incorrect. Whether the null hypothesis is correct or not is not random. We just don't know the truth. We can only say that the probability of making a correct rejection is 0.95.

(c) Correct.

(d) Incorrect. The population mean is a fixed parameter but not a random variable. It can either be within [-18.1, 23.4] or not within this interval. We just don't know exactly its value. We can only say that based on the procedure of constructing the interval, we will have 0.95 probability to obtain an interval that will contain the population mean.

(e) Correct.

(f) Incorrect. Double the sample size will make the length of the confidence interval become ¼ of the length of the original interval.

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