# MATH 246, Midterm Examination, Fall 1998

(1) (**30 pts**): A continuous random variable $X$ has pdf

$$f_X(X) = \begin{cases} C(1-x^4) & -1 \leq \text{X} \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Find $C$.

(b) Find the cdf of X.

(c) Find $P[|X| < \frac{1}{2}]$.

(d) Find $E[X]$ and $E[X^2]$

(2) (**30 pts**): Let $(X, Y)$ have joint pdf

$$f_{X,Y} = \begin{cases} ye^{-y(1+x)} & x > 0,\ y > 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the marginal pdf of $X$ and of $Y$.

(b) Are $X, Y$ independent?

(c) Find the probability $P(X \leq 1, Y \leq 1)$.

(d) Find the probability $P(X + Y \leq 2)$.

(2) (**40 pts**): The three balls numbered 1 to 3 in an urn are drawn at random one at a time until the urn is empty. The sequence of the ball numbers is noted

(a) Find the sample space.

(b) Find the set $A_k$ corresponding to the events "ball number k is selected in the kth draw" for $k = 1, 2$.

(c) Are $A_1, A_2$ independent?

(d) Find $P(A_1 \cap A_2)$

(e) Defined the random variable $X$ to be the summation of the numbers on the first and second balls. Find the probability mass function for $X$.