COMP 272: Final Exam
2000 Spring Semester

Name: ______________________ E-mail: ______________________
Student No. ______________________ Tutorial: ______________________

You have 3 hours to solve 7 problems for a total of 100 marks. Answer all problems in the space provided. You may also use the back of the test papers. Good luck!

1	
2	
3	
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5	
6	
7	
Total	

Problem 1. (20 points)

For each statement below, circle the appropriate answer and give a short explanation of your answer in each case.

a) $\{a^n b^n : n \geq 0\} \circ \{b^n c^n ; n \geq 0\} = \{a^n b^{2n} c^n : n \geq 0\}$

True False

Explanation:

b) Let A be a set of languages. If A is closed under intersection and complementation, then A is closed under union.

True False

Explanation:

c) The deterministic pushdown automaton is equivalent to the nondeterministic pushdown automaton.

True False

Explanation:

d) If L_1 is nonregular, L_2 is finite, then $L_1 - L_2$ is nonregular.

True False

Explanation:

e) The set of all non-recursively enumerable languages over $\{0, 1\}$ is countable.

True False

Explanation:

f) There exists some Turing machine M_0 for which the following problem is decidable: Given an arbitrary w, does M_0 halt on w?

True False

Explanation:

Problem 2. (10 points)

a) (5 points)
Construct a deterministic finite automaton that accepts

$$\{w : w \in \{0,1\}^* \text{ and } w \text{ has three consecutive 0's}\}$$

b) (5 points)
Give a regular expression for the following language over $\{0,1\}$

$$\{w : w \text{ contains both the substring 11 and the substring 010}\}$$

Problem 3. (15 points)

a) (3 points)
State the pumping theorem for regular languages.

b) (12 points)
Using the Pumping Theorem you just stated, prove that the following language is not regular:

$$L = \{ab^i c^j a : 0 \leq i < j\}$$

Problem 4. (10 points)

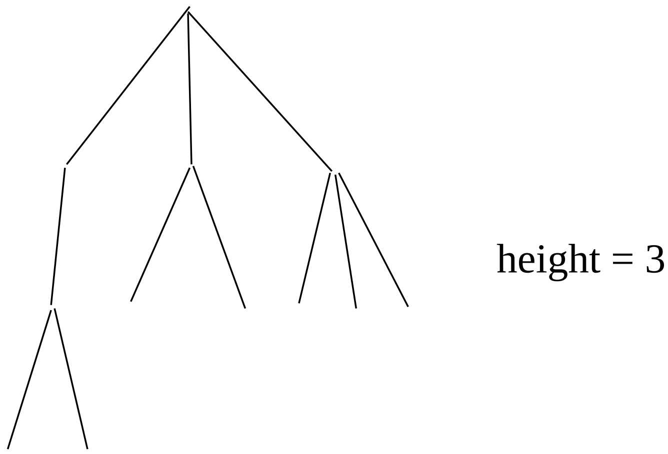
a) (6 points)
Given a CFG G with rules $S \to S + S$, $S \to S * S$, $S \to (S)$, $S \to a$, Construct a pushdown automaton that accepts $L(G)$.

b) (2 points)
Trace a sequence of computation moves by which the string $a * (a + a)$ is accepted by your pushdown automaton. Show the configuration at each step (*i.e.* the state, the unread input, and the stack contents.).

c) (2 points)
Give the leftmost derivation of the string $a * (a + a)$.

Problem 5. (15 points)

Let $G = (V, \Sigma, R, S)$ be a context-free grammar. The *fanout* of G, denoted by $\phi(G)$, is the largest number of symbols on the right-hand side of any rule in R. The *height* of a parse tree T is the number of line segments in the longest path from the root to some leaf in T.



a) (3 points)
For any parse tree T of grammar G of height h, what is the maximum length of the yield of T?

b) (3 points)
What is the minimum height h of any parse tree T of G such that T has at least one nonterminal repeated in its longest path?

c) Consider the grammar $G = (V, \Sigma, R, S)$ where $V = \{S, +, *, (,), a\}$, $\Sigma = \{+, *, (,), a\}$, $R = \{\ S \to S + S,\ S \to S * S,\ S \to (S),\ S \to a\}$

i) (1 point)
What is the *fanout* of G?

ii) (8 points)
Show that for $w = a{*}(a{+}a)$, there exist $u, v, x, y, z \in \Sigma^*$ such that $w = uvxyz$, $vy \neq e$, and $uv^ixy^iz \in L(G)$ for $i = 0, 1, 2, \cdots$. (You must give the value of the substrings u, v, x, y, and z.)

Problem 6. (15 points)

Prove that the following problem is undecidable: Given a Turing machine M, is $L(M)$ context-free?

Problem 7. (15 points)

a) (10 points)
Show that the following language is recursively enumerable.

$$L_1 = \{\text{“}M\text{”} : M \text{ is a Turing machine and } L(M) \neq \emptyset\}.$$

b) (5 points)

We have proved in class that the following language is not recursive.

$$L_2 = \{\text{“}M\text{”} : M \text{ is a Turing machine and } L(M) = \emptyset\}$$

Is it recursively enumerable? Give a brief explanation. (Hint: use the fact that L_1 in part (a) is recursively enumerable)