COMP 272 Theory of Computation
Lecture 1 – Dr. Mordecai Golin
December 16, 1996 Final Exam

- Answer **ALL** questions. The exam is worth 400 points.
- Time allowed: 180 minutes.
- Write your answers in the test book.

1. (**60 pts**) In your own words briefly define the following terms/symbols (two to three sentences each will suffice).

   (a) $\mathcal{P}$.

   (b) $\mathcal{NP}$.

   (c) Turing Decidable Languages.

   (d) Universal Turing Machines.

2. (**60 pts**) Let

$$K_0 = \{\rho(M) \ : \text{ Turing Machine } M \text{ accepts input string } \rho(M)\}$$

   Prove that K_0 is not Turing Decidable.

3. (**100 pts**) Prove the truth or falsehood of the following statements. In your proofs completely state the theorems you are using (their assumptions and conclusions).

   (a) If L is Turing Decidable then $\overline{L}$ is Turing Acceptable.

   (b) There are languages in $\mathcal{P}$ that are not in $\mathcal{NP}$.

   (c) If L is a Context Free Language and F is a finite language then $(L - F)$ is a Context Free Language.

   (d) If L is not a Context Free Language and F is a finite language then $(L - F)$ is not a Context Free Language.

4. (**60 pts**) Construct a Turing Machine that *accepts* the language

$$L = \{uu^R \, : \, u \in \{a,b\}^*\}.$$

That is, your Turing Machine should start in configuration $(s, \#w\underline{\#})$, where $w \in \{a,b\}^*$ contains no blanks, and halt if and only if $w \in L$. By *construct* we mean draw a Turing machine by combining smaller Turing machines in the fashion described in class. The smaller Turing machines you may use are

L :	Move left one square
R :	Move Right one square
L_σ :	Move left until a σ is found
R_σ :	Move right until a σ is found
W_σ or σ :	Write the character σ

(σ signifies any character in $\{a,b\}$). Note: u^R is the reverse of u.

5. (**60 pts**) Let

$$L = \{w \in \{a,b\}^* \; : \; \text{number of } a\text{-s in } w = (\text{number of } b\text{-s in } w)^2\}$$

Is L Context-Free? Is L Regular? Prove your answer. When proving your answer fully state the theorems that you are using (including their assumptions and their conclusions).

6. (**60 pts**) Let

$$L = \{w \in \{a,b,c\}^* \; : \quad \begin{array}{rl} & \text{the length of } w \text{ is odd} \\ \text{and} & \text{the middle symbol of } w \text{ is an } a\} \end{array}$$

Is L Context-Free? Is L Regular? Prove your answer. When proving your answer fully state the theorems that you are using (including their assumptions and their conclusions).