COMP 272/L1: Final Exam
1998 Spring Semester – Sunil Arya

Name: ____________________ E-mail: ____________________
Student No. ____________________ Tutorial: ____________________

You have 2 hours 30 minutes to solve 7 problems for a total of 100 marks. Answer all problems in the space provided. You may also use the back of the test papers. Good luck!

Problem 1. (20 points)

For each statement below determine whether it is true or false. Circle the appropriate answer and give a short explanation of your answer in each case.

Note: You will receive no credit unless your explanation is correct.

(a) If a DFA A has a final state, then the language accepted by A is not the empty set.

True False

Explanation:

(b) If L_1 is a context-free language and L_2 is a finite language, then $L_1 - L_2$ must be a context-free language.

True False

Explanation:

(c) Any language corresponding to a regular expression that uses only the union and concatenation operation, but does not involve the Kleene star operation is finite.

True False

Explanation:

(d) Let L be the language of strings over the alphabet $\{0, 1\}$ that are binary representations of prime numbers. Then L is recursive.

True False

Explanation:

(e) Suppose L is decided by a Turing machine M that moves its tape head right on each move. Then L must be regular.

True False

Explanation:

(f) There is at least one Turing machine M_0 for which the following problem is undecidable: Given w, does M_0 halt on w?

True False

Explanation:

(g) The complement of a recursively enumerable language must be recursively enumerable.

True False

Explanation:

Problem 2. (10 points)

Find a regular expression corresponding to each of the following languages over $\{0, 1\}$. No explanation is needed.

(a) The language of strings that do not end with 01.

(b) The language of strings that do not contain the substring 110.

Problem 3. (10 points)

(a) (2 points)
State the pumping theorem for regular languages.

(b) (8 points)
Using the pumping theorem you just stated, prove that the following language is not regular:

$$L = \{w \in \{a, b\}^* : w = w^R\}.$$

Here w^R denotes the reverse of the string w. For example, $(abbb)^R = bbba$

Problem 4. (15 points)

Consider the context-free grammar $G = (V, \Sigma, R, S)$, where $V = \{a, b, S, A, B\}$, $\Sigma = \{a, b\}$, and

$$R = \{S \to AB, \quad S \to BA,$$

$$A \to aAa, \quad A \to aAb, \quad A \to bAa, \quad A \to bAb, \quad A \to a,$$

$$B \to aBa, \quad B \to aBb, \quad B \to bBa, \quad B \to bBb, \quad B \to b\}$$

(a) Describe, in plain English, the set of strings that can be generated, starting with the nonterminal A? Also, describe the set of strings that can be generated, starting with the nonterminal B?

(b) Let *Not-Double* be the language of strings over alphabet $\{a, b\}$ which are of the form uv where u and v are strings of equal length and u is not equal to v. Show that *Not-Double* is exactly the set of strings of the form $xaybz$ or $xbyaz$ where x is a string of length i, z is a string of length j, and y is a string of length $i + j$ for some $i, j \geq 0$.

(c) The grammar G above generates the language *Not-Double*. Explain why this is true.

Note: An informal explanation will suffice; you do not need to give a rigorous proof by induction.

Problem 5. (15 points)

Let L denote the set of strings of properly balanced left and right parenthesis. For example, $()(()) \in L$, $(()() \notin L$.

(a) (7 points)

Construct a one-tape Turing machine to decide language L. You may assume that the initial configuration is $\triangleright\underline{\sqcup}w$, where w is a string consisting only of left and right parenthesis.

Note: You need to draw the Turing machine by combining the simpler machines (e.g. symbol-writing and head-moving machines and the machines y and n), as described in class.

(b) (8 points)

Describe in plain English the operation of the Turing machine you designed in part (a).

Problem 6. (15 points)

Prove that the following problem is undecidable: Given a Turing machine M, does M halt on *exactly* three input strings?

Note: You are allowed to use the following fact: the halting problem for Turing machines is undecidable. Do *not* assume the undecidability of any other problem.

Problem 7. (15 points)

Let

$$L = \{\text{“}M\text{”} : M \text{ halts on } \textit{at least} \text{ three input strings}\}.$$

Is language L recursively enumerable? Fully justify your answer.