COMP 272: Final Exam
1999 Fall Semester

Name: ____________________ E-mail: ____________________
Student No. ____________________ Tutorial: ____________________

You have 2 hours 30 minutes to solve 8 problems for a total of 100 marks. Answer all problems in the space provided. You may also use the back of the test papers. Good luck!

1	
2	
3	
4	
5	
6	
7	
8	
Total	

1) (15 points)

For each statement below determine whether it is true or false. Circle the appropriate answer. No explanation is needed.

a) The language $\{a^i b^j c^k : i, j, k \geq 0\}$ is context-free.

True False

b) The language $\{a^i b^j c^k : 0 \leq i + j \leq k\}$ is context-free.

True False

c) The language $\{a^i b^j c^k : i \leq j \text{ and } j \geq k\}$ is context-free.

True False

d) If L is context-free and R is regular, then $L - R$ is context-free.

True False

e) If L is context-free and R is regular, then $R - L$ is recursive.

True False

f) The following problem is undecidable:
Given a language L that is recursive and an input string w, is $w \in L$?

True False

g) The following problem is undecidable:
Given the universal Turing machine U and an input string w, does U accept w?

True False

2) (10 points)

a) (5 points)
Find a regular expression for the following language:

$$\{w \in \{0,1\}^* : w \text{ contains no more than one occurrence of the string } 00\}$$

(The string 000 should be viewed as containing two occurrences of 00.)

b) (5 points)
Draw a deterministic finite automaton that accepts the language in (a). (You do not have to give the 5-tuple representation.)

3) (10 points)

a) (2 points)
State the pumping theorem for regular languages.

b) (8 points)
Prove that the following language is not regular using the pumping theorem stated above.

$$\{0^n w | w \in \{0,1\}^* \text{ and } |w| \leq n\}$$

4) (15 points)

a) (8 points)
Give a context-free grammar that generates the language

$$L = \{a^i b^j c^k : 0 \le i + k \le j\}$$

b) (7 points)
Consider the pushdown automaton $M = (K, \Sigma, \Gamma, \Delta, s, F)$ where

$$\begin{aligned}
K &= \{s, q, f\}, \\
F &= \{f\}, \\
\Sigma &= \{a, b\}, \\
\Gamma &= \{a\}, \\
\Delta &= \{ ((s, a, e), (s, a)), ((s, b, e), (s, a)), ((s, b, e), (q, e)), ((q, a, a), (q, e)), \\
&\qquad ((q, b, a), (q, e)), ((q, e, a), (f, e)), ((f, e, a), (f, e))\}
\end{aligned}$$

Describe $L(M)$ in English.

5) (12 points)

Describe a standard Turing machine that decides the following language

$$L = \{a^i b^j c^k : 0 \leq i \leq j \leq k\}$$

You may assume that the initial configuration is $\rhd\underline{\sqcup}w$, where $w \in \{a, b, c\}^*$. You have to draw the Turing machine by combining the basic machines (e.g. symbol-writing and head-moving machines and the machine y and n), as described in class. Trace the operations of your machine when started on $\rhd\underline{\sqcup}abbccc$.

6) (12 points)

We have proved that the set of recursively enumerable languages is closed under union and intersection. We let L_1 and L_2 be two recursively enumerable languages over Σ and let M_1 and M_2 be Turing machines accepting L_1 and L_2, respectively, and construct a two-tape Turing machine that simulates both M_1 and M_2 *simultaneously*. Consider instead the idea of simulating M_1 and M_2 *sequentially*, which can easily be done using only one tape, as follows. First, the tape $\triangleright\underline{\sqcup}w$ is tranformed into $\triangleright\underline{\sqcup}w\#w$. Then M_1 is simulated, using the second copy of w as input and using the marker $\#$ as the left-end of the tape. If and when M_1 halts, the second part of the tape is erased (i.e. the tape is now $\triangleright\underline{\sqcup}w$), and M_2 is simulated.

a) Could such a Turing machine be used to show that the union of two recursively enumerable languages is recursively enumerable? Why?

b) Could such a Turing machine be used to show that the intersection of two recursively enumerable languages is recursively enumerable? Why?

7) (10 points)

Prove that the following problem is undecidable. Given two Turing machines M_1 and M_2, is $L(M_1) \subseteq L(M_2)$? (You may wish to reduce from the Halting problem or the problem "Given a Turing machine M, is $L(M) = \emptyset$?")

8) (16 points)

a) (8 points)
Is the following problem solvable? Justify your answer.
Given a deterministic Turing machine M, does there exist a string w such that M accepts w in *less than* 10 steps (write or move)?

b) (8 points)

Is the following problem solvable? Justify your answer.
Given a deterministic Turing machine M, does there exist a string w such that M accepts w in *at least* 10 steps?