COMP 272 Theory of Computation
November 29, 1996 **Exam One**

Name: ____________________ E-mail: ____________________
Student No: ____________________ Lecture: ____________________
Tutorial: ____________________

- Answer **ALL** questions. The exam is worth 300 points.
- Time allowed: 80 minutes.

1. **DFAs and Regular Grammars (70 points)**

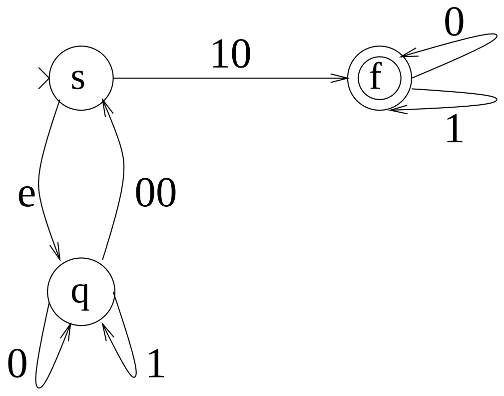
   Let
   $$L = \{w \in \{a, b\}^* : \; w \textit{ does not } \text{contain } aab \text{ as a substring}\}.$$

   (a) Construct a DFA that accepts L.

   (b) Construct a regular grammar G that generates L (i.e., $L(G) = L$). Recall that a regular grammar is a special form of a context-free grammar.

2. **NFAs and DFAs (60 points)**

Build a DFA that accepts the same language as that accepted by the following NFA:



3. **Regular Languages (90 points)**

Let

$$L = \{w \in \{a, b\}^* : \text{ number of } a\text{-s in } w \leq \text{number of } b\text{-s in } w\}$$

Is L regular? Prove your answer. When proving your answer fully state the theorems that you are using (including their assumptions and their consequences).

4. **Context Free Languages and Pushdown Automata (80 points)**

(a) Write down a Context Free Grammar that generates the following language:

$$L = \{w \in \{a, b, c\}^* : \text{ number of } a\text{-s in } w = \text{number of } b\text{-s in } w\}$$

(b) Build a pushdown automaton that recognizes L. Recall that a pushdown automaton is of the form

$$M = (K, \Sigma, \Gamma, \Delta, s, F)$$

where Δ, the transition relation, is of the form

$$\Delta \subset (K \times \Sigma^* \times \Gamma^*) \times (K \times \Gamma^*).$$

In this case $\Sigma = \{a, b, c\}$ and you must specify the remaining components of M.