COMP 272/L1: Mid-term Exam
1998 Spring Semester – Sunil Arya

Name: ____________________ E-mail: ____________________
Student No. ____________________ Tutorial: ____________________

You have 2 hours to solve 7 problems for a total of 100 marks. Answer all problems in the space provided. You may also use the back of the test papers. Good luck!

Problem 1. (15 points)

For each statement below determine whether it is true or false. Circle the appropriate answer. All parts refer to languages over $\{a, b\}$.

Note: No explanation is needed.

(a) Every subset of a regular language over $\{a, b\}$ is regular.

True False

(b) The language $\{w \in \{a, b\}^* : w = w^R\}$ is regular.

True False

(c) If L is a regular language over $\{a, b\}$, then $\{w : w^R \in L\}$ is also regular.

True False

(d) If C is any finite set of regular languages over $\{a, b\}$, then the union of all the languages in C is also regular.

True False

(e) If L is not regular, then $\overline{L}$, the complement of L, is also not regular.

True False

(f) The language $\{ww : w \in \{a\}^* \}$ is regular.

True False

(g) The language $\{ww : w \in \{a, b\}^* \}$ is regular.

True False

(h) The language $\{wbw : w \in \{a\}^* \}$ is regular.

True False

(i) The language $\{wbw : w \in \{a\}^* \}$ is context-free.

True False

(j) The language $\{a^{2n} : n \geq 0\}$ is regular.

True False

Problem 2. (15 points)

Let $\mathcal{N}$ denote the set of natural numbers. Prove from scratch that the set of all functions from the set $\mathcal{N}$ to $\{0, 1\}$ is uncountable.

Note: Do not use any fact proved in class.

Hint 1: Use the diagonalization technique.
Hint 2: Two functions f_1 and f_2 are considered different if and only if $f_1(i) \neq f_2(i)$ for some $i \in \mathcal{N}$.

Problem 3. (15 points)

(a) (5 points) Construct a DFA accepting the following language:

$$L = \{w \in \{a, b\}^* : \text{there is an } a \text{ immediately before each } b \text{ in } w\}.$$

For example, $abaaba \in L$, but $b \notin L$ and $aabba \notin L$.
Note: No explanation is needed.

(b) (5 points)

Write a regular expression for the language in part (a).

Note: No explanation is needed.

(c) (5 points)

Using the $\vdash$ function defined in class, write out the computation sequence for your DFA when the input is *abaab*.

Problem 4. (10 points)

Construct an NFA that accepts the language: $((ab \cup bb)^* aa)^*$.

Note: There is no need to try to reduce the number of states in your NFA.

Problem 5. (20 points)

(a) (5 points)
State the pumping theorem for regular languages.

(b) (10 points)
Using the pumping theorem you just stated, prove that the following language is not regular:

$$L_1 = \{01^p01^q : p = q,\ p \geq 0, q \geq 0\}.$$

(c) (5 points)

Let

$$L_2 = \{01^p01^q : p \neq q,\ p \geq 0, q \geq 0\}.$$

Prove that L_2 is not regular by using the fact that language L_1, given in part (a), is not regular.

Note: You may use any fact proved in class, as long as you state it clearly.

Problem 6. (15 points)

(a) (10 points)
Find a context-free grammar for the following language:

$$\{a^i b^j c^k : i = j + k\}.$$

Note: No explanation is needed.

(b) (5 points)
Draw a parse tree for *aaaabbcc* using the grammar of part (a). The root of the parse tree must be labelled by the start symbol of your grammar.

Problem 7. (10 points)

Find a context-free grammar for the following language:

$$\{a^i b^j : i \leq 2j\}.$$

Note: No explanation is needed.