COMP272 Midterm Suggested Solution

(1) (a) False
(b) False
(c) True
(d) True
(e) True
(f) True
(g) False
(h) False
(i) True
(j) True

(2) Proof:
Let F be the set of all functions from the set N to $0, 1$.
Suppose that F is countably infinite. That is, we assume that that there is a way of enumerating all members of F as

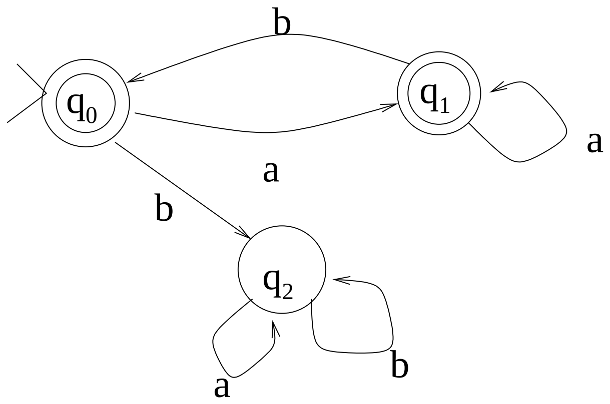
$$F = \{f_0, f_1, f_2, \cdots\}$$

Now consider the function

$$d(i) = \begin{cases} 0 & \text{if } f_i(i) = 1 \\ 1 & \text{if } f_i(i) = 0 \end{cases}$$

Since d is a function from the set N to $\{0, 1\}$, it should appear somewhere in the enumeration $\{f_0, f_1, f_2, \cdots\}$ But d cannot be f_0, because it differs from f_0 with respect to $f_0(0) \neq d(0)$; and it cannot be f_1 because it differs from f_1 with respect to $f_1(1) \neq d(1)$; and so on. We must conclude that d does not appear on the enumeration at all, and this is a contradiction.
We arrived at this contradiction starting from the assumption that F is countably infinite; we must therefore conclude that this assumption was in error. Hence F is uncountable.

(3a)

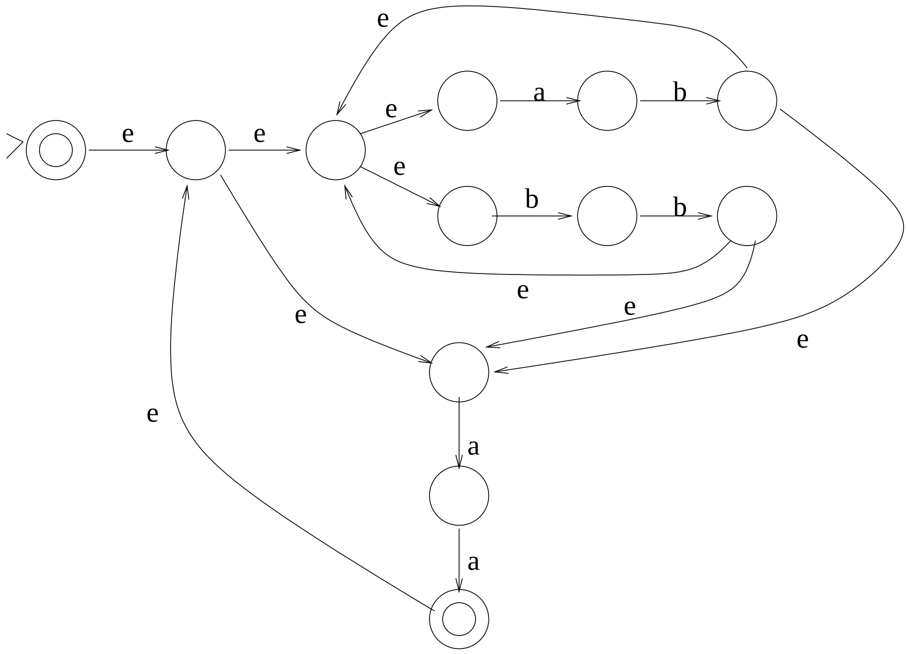


(3b) $(aa^*b)^*a^*$
Another correct solution is: $(a \cup ab)^*$

(3c)

$$\begin{aligned}(q_0, abaab) &\vdash_M (q_1, baab)\\ &\vdash_M (q_0, aab)\\ &\vdash_M (q_1, ab)\\ &\vdash_M (q_1, b)\\ &\vdash_M (q_0, e)\end{aligned}$$

(4)



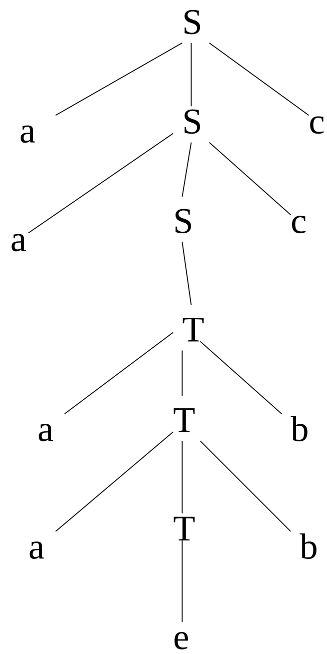
(5a) Let L be a regular language. There is an integer $n \geq 1$ such that any string $w \in L$ with $|w| \geq n$ can be rewritten as $w = xyz$ such that $y \neq e$, $|xy| \leq n$, and $xy^iz \in L$ for each $i \geq 0$.

(5b) The language $L = \{01^p01^q : p = q, p \geq 0, q \geq 0\}$ is not regular, for if it were regular, the pumping theorem would apply for some integer n. Consider then the string $w = 01^n01^n \in L$. By the theorem, it can be rewritten as $w = xyz$ such that $|xy| \leq n$ and $y \neq e$ – that is, $y = 01^i$ or $y = 1^i$ for some $i \geq 0$. In each of these 2 cases, we get a contradiction, as follows. If $y = 01^i$, $xz = 1^{n-i}01^n \notin L$; and if $y = 1^i$, $xz = 01^{n-i}01^n \notin L$,

(5c) Let $L_3 = L(01^*01^*)$ where L_3 is regular. ASSUME L_2 is regular. We can observe that $L_1 = L_3 - L_2 = L3 \cap L_2'$ where L_2' is the complement of L_2. Since regular languages are closed under complementation, it follows that L_2' is regular. Further, since regular languages are also closed under intersection. it follows that L_1 is regular, which is a contradiction. Therefore, our assumption must be incorrect, that is, L_2 is not regular.

(6a) $G = \{V, \Sigma, R, S\}$
$V = \{a, b, c, S, T\}$
$\Sigma = \{a, b, c\}$
$R = \{S \to aSc, S \to T, T \to aTb, T \to e\}$

(6b)



(7) $G = \{V, \Sigma, R, S\}$
$V = \{a, b, S\}$
$\Sigma = \{a, b\}$
$R = \{S \to Sb, S \to aSb, S \to aaSb, S \to e\}$