COMP 272 Theory of Computation

October 29, 1999, **Midterm Examination**

Name: ______________________ E-mail: ______________________

Student No: ______________________ Tutorial: ______________________

You have 2 hours to solve 7 problems for a total of 100 points. Answer all problems in the space provided. You may also use the back of the test papers. Good luck!

1) (12 points)
For each statement below determine whether it is true or false. Circle the appropriate answer. No explanation is needed.

a) Any language represented by a regular expression that does not involve the Kleene star operation is finite.

True False

b) There is a language L over a finite alphabet such that L is uncountable.

True False

c) The set of all non-regular languages is uncountable.

True False

d) If L is regular and $L_1 \subseteq L$, then L_1 is regular.

True False

e) If L is not regular and $L_1 \subseteq L$, then L_1 is also not regular.

True False

f) The language $\{w \in \{a, b\} : w \neq w^R\}$ is regular.

True False

g) The language $\{wbw : w \in \{a, b\}^*\}$ is context free.

True False

h) The language $\{a^{6n+4} : n \geq 0\}$ is not regular.

True False

2) (5 points)
For each statement below, fill in *countable* (C), *uncountable* (U), or *could be countable or uncountable* (C/U).

1. If set A is uncountable and set B is countable,

   then $A \cap B$ is ____________.

2. If set A is uncountable and set B is countable,

   then $A \setminus B$ is ____________.

3. If set A is uncountable and set B is uncountable,

   then $A \cap B$ is ____________.

4. If set A is uncountable and set B is uncountable,

   then $A \cup B$ is ____________.

3) (15 points)

a) (8 points)
Construct a DFA that accepts the following language:

$$L = \{w \in \{a, b\}^* : w \text{ has an odd occurrence of the substring } abb\}$$

For example, $babb, abbbaabbabb \in L$.

b) (2 points)
Write out the computation sequence for your DFA when the input string is $ababb$.

c) (5 points)
Write a regular expression for the language in part (a).

4) (18 points)
Prove or disprove the following claims:

a) If L_1 is regular and L_2 is non-regular, then $L_1 L_2$ is non-regular.

b) If L and L_1 are regular and $L = L_1 \cup L_2$, then L_2 is regular.

c) The set of regular languages is closed under countably infinite union (i.e., $L_1 \cup L_2 \cup L_3 \cup \ldots.$)

5) (14 points)

a) (8 points)
Draw an NFA or a DFA that accepts the language

$$\{w \in \{a, b\}^* : w \neq ab \text{ and } w \neq aba\}$$

b) (6 points)
Write a regular expression for the language in part (a)

6) (22 points)

a) (12 points)
Use the pumping theorem to prove that the following language is not regular:

$$\{w \in \{a, b\}^* : n_a\text{= number of } a\text{'s in } w, n_b\text{= number of } b\text{'s in } w,\ \text{ and } |n_a - n_b| \leq 2\}$$

b) (10 points)
Prove that $L = \{a^i b^j : i \neq j, \text{ and } i, j \geq 0\}$ is not regular without using the pumping theorem. You may use any fact proved in class (without proving it).

7) (14 points)

a) (10 points)
Write a context-free grammar that generates the following language:

$$L = \{a^i b^j c^k : i = j \text{ or } i = k\}$$

Note that the *or* is an inclusive or, i.e., all three of the following possibilities are allowed: (i) $i = j$, (ii) $i = k$, (iii) $i = j$ and $i = k$.

b) (4 points)
Draw a parse tree for $aabbcc$ using your grammar in part (a).